AN INVESTIGATION OF THE DISTRIBUTION OF STRESS OVER WIDE REINFORCED CONCRETE BEAMS UNDER CONCENTRATED LOADS

BY

LYMAN LIONEL LIVINGSTON

THESIS

FOR THE

DEGREE OF BACHELOR OF SCIENCE

IN

CIVIL ENGINEERING

COLLEGE OF ENGINEERING

UNIVERSITY OF ILLINOIS

1912

UNIVERSITY OF ILLINOIS

COLLEGE OF ENGINEERING.

May 24, 1912

This is to certify that the thesis of LYMAN LIONEL LIVINGSTON entitled AN INVESTIGATION OF THE DISTRIBUTION OF STRESS OVER WIDE REINFORCED CONCRETE BEAMS UNDER CONCENTRATED LOADS was prepared under my personal supervision; and I recommend that it be approved as meeting this part of the requirements for the degree of Bachelor of Science in Civil Engineering.

W A Slater

A N Talbot

Instructors in Charge.

Recommendation approved:

Ira O. Baker

Professor of Civil Engineering.

CONTENTS

I. Introduction

1. Purpose.- The purpose of this thesis is to investigate the effect of concentrated loads on wide beams, and to determine the amount of transverse reinforcement required in a beam of given dimensions, in order to have the longitudinal and transverse steel fail at the same time. Tests were made in 1910, and 1911, at the University of Illinois for the same purpose. The tests of the 1910 series were made by K.E.Robinson and are reported in his thesis entitled, "Tests of Reinforced Concrete Beams: Effect of Lateral Distribution of Loads." The tests made in 1911 were by E.J.Schell and are reported in his thesis entitled, "The Effect of Lateral Distribution of Loads over Wide Reinforced Concrete Beams." Their tests were made on beams of different proportions between lengths, widths, and thickness, and with different proportions of the widths loaded.

In the beams of this thesis, the widths, and the percentages of longitudinal and transverse reinforcement were the variables. The span was 48 inches and the depth was six inches in all cases with the exception of beam No. 750.1. Its depth was made only three inches in order to insure tension failure in the steel, and to afford a basis for direct comparison with the results of the previous tests. In all three series the load was applied at the one-third points of the span. The only exceptions to this are beams No. 749.1 and 749.2 of the 1911 series. In planning each series of tests an attempt was made to design the test beams of one series so that the tests would be similar in as many respects as possible, to those of the preceding

series, in order to afford a direct method of comparing results.

2. Supervision.- The tests described in this thesis were made under the direction of the Illinois Engineering Experiment Station staff, and form a part of a more elaborate investigation of reinforced concrete beams. The interpretation of results and the conclusions drawn are original with the writer of this thesis.

3. Acknowledgment.- Mr. T.G.Burgess, a senior student of the University of Illinois, assisted in the greater part of the testing work, and it was to have been a part of his thesis, but owing to his sickness and death before the testing was finished, the work was completed by the writer, with the assistance of men engaged by the Experiment Station staff to do the mechanical work.

II. Materials, Test Prices, and Method of Testing.

4. Materials.- The sand and stone were ordered under the same specifications as those given in the bulletins of the Engineering Experiment Station.

Stone.- The stone was a good quality of limestone from Kankakee, Illinois, ordered screened through a one-inch and over a one-quarter inch screen, and was graded as shown in Table I. page 3.

Sand.- The sand used came from near the Wabash river at Attica, Indiana. It was relatively free from dirt and was graded as shown in Table II. page 3 .

Table I

Mechanical Analysis of Stone.

Average of 5 samples

Size of Square Opening	Separation Size Inches	Per Cent Passing
1 in.	--	100.
3/4 "	--	95.5
1/2 "	--	66.7
3/8 "	--	46.3
No.3	0.28	25.9
No.5	0.174	8.1
No.10	0.091	3.4

Table II

Mechanical Analysis of Sand.

Average of 5 samples

Sieve No.	Separation Size Inches	Per Cent Passing
3	0.28	100.
5	.174	88.
10	.091	54.3
12	.067	47.5
16	--	41.7
18	.043	32.9
30	.027	21.2
40	.019	13.3
50	.013	5.1
74	.009	2.7
150	--	1.0

Cement.- Universal Portland Cement was used. Tests on this cement showed the initial set to occur at 3 hr. 5 min. and final set at 6 hr. 32 min. after mixing. Sieve tests showed 97.2% passing a No.100 sieve and 81.8% passing a No.200 sieve. The test of the briquettes are shown in Table III. These tests were made according to standard methods by Mr. B.L.Bowling at the Cement Testing Labratory, University of Illinois.

Table III

Briquette Test of Universal Portland Cement.

Each value is the average of five tests.

Loads are given in lb. per. sq. in.

Sample No.	Date	Neat 7 Days	Cement 28 days	1 - 3 7 Days	Mortar 28 Days
1	Oct. 25, 1911	585	685	239	315
2	Nov. 11, 1911	577	694	225	297
3	Dec. 7, 1911	691	715	242	306
4	Dec. 22, 1911	617	792	231	326
5	Jan. 10, 1912	588	672	246	333
6	Feb. 12, 1912	612	758	253	323
7	Feb. 28, 1912	698	884	287	372
	Average	624	743	246	325

Concrete.- A 1-2-4 mixture was used for all the specimens. Men in the employ of the Experiment Station, who were accustomed to making the concrete mixed the materials and made the test beams. The mixing was done with shovels by hand.

Steel.- All the steel used was mild steel, and came from the Illinois Steel Company. The yield point was at a stress of about 42,000 lb. per. sq. in.

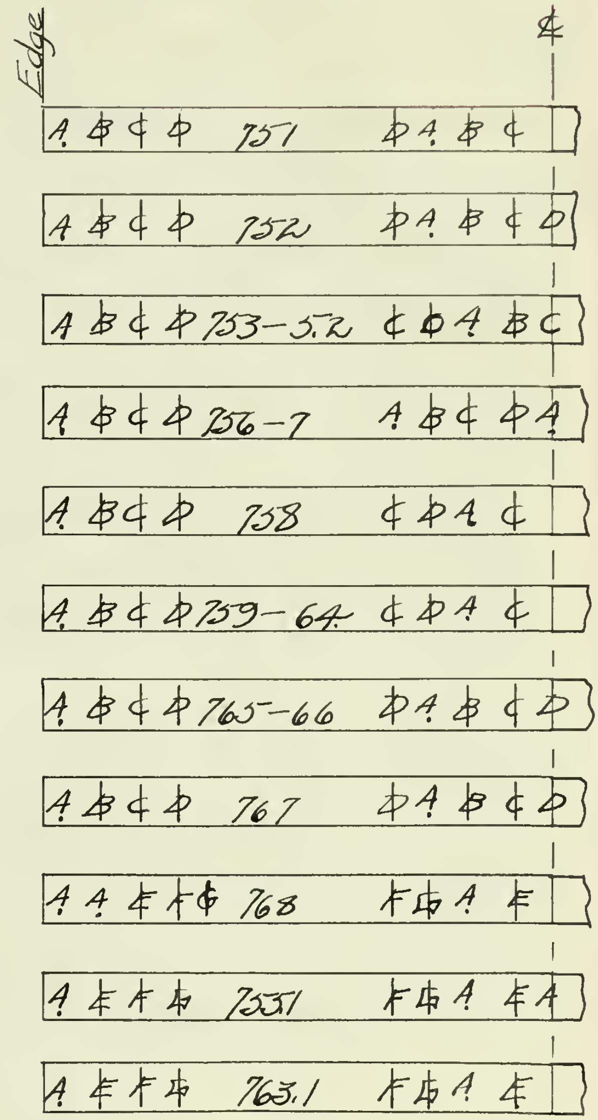
5. Test Beams.- The summary sheet page 16 gives the dimensions of the test beams and the amounts of reinforcement used. Page 5 shows how the rods were placed in the beams, and the lengths of the different bent up rods.

6. Method of Making the Beams.- The beams were made directly on the concrete floor of the laboratory, a strip of building paper being laid beneath the forms to prevent the concrete adhering to the floor. The forms used were the ordinary wooden knock-down type.

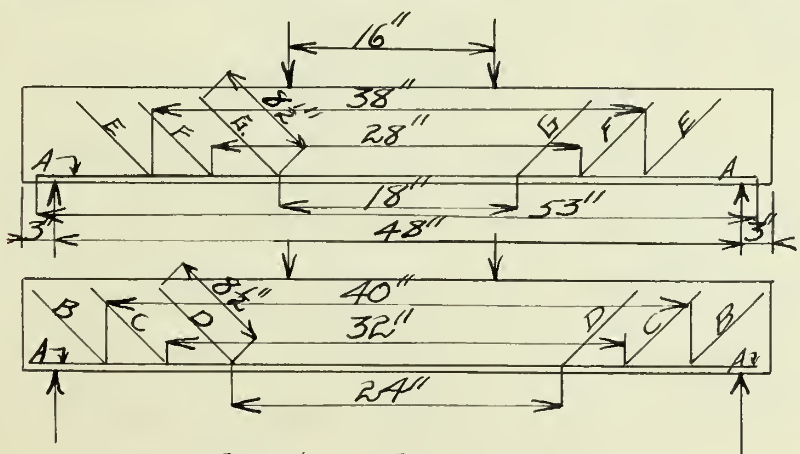
7. Storage.- The beams were stored in a room the temperature of which was from 60 F. to 70 F. They were tested

Sketches Showing Spacing of Rods in beams.

Beam No.	Size of Rods	Type of Rod A. No.	B No.	C No.	D. No.
750.	3/8	14.	0	0	0
751.	"	4	4	4	2
752.	7/16	4	4	4	3
753.	"	8	8	7	6
754.	"	"	"	"	"
755.2	"	"	"	"	"
756.	1/2	9	8	8	8
757.	"	"	"	"	"
758.1	"	11	11	11	11
759.	3/8	6	4	4	6
760.	"	"	"	"	"
761.	"	"	"	"	"
762.	"	"	"	"	"
763.2	7/16	"	"	"	"
764.	"	"	"	"	"
765.	1/2	6	6	6	5
766.	"	"	"	"	"
767.	7/16	10	10	10	9
768.1	3/8	8	E 4	F 6	E 4
755.1	7/16	9	8	6	6
763.1	"	6	6	4	4



End View



Side Elevation

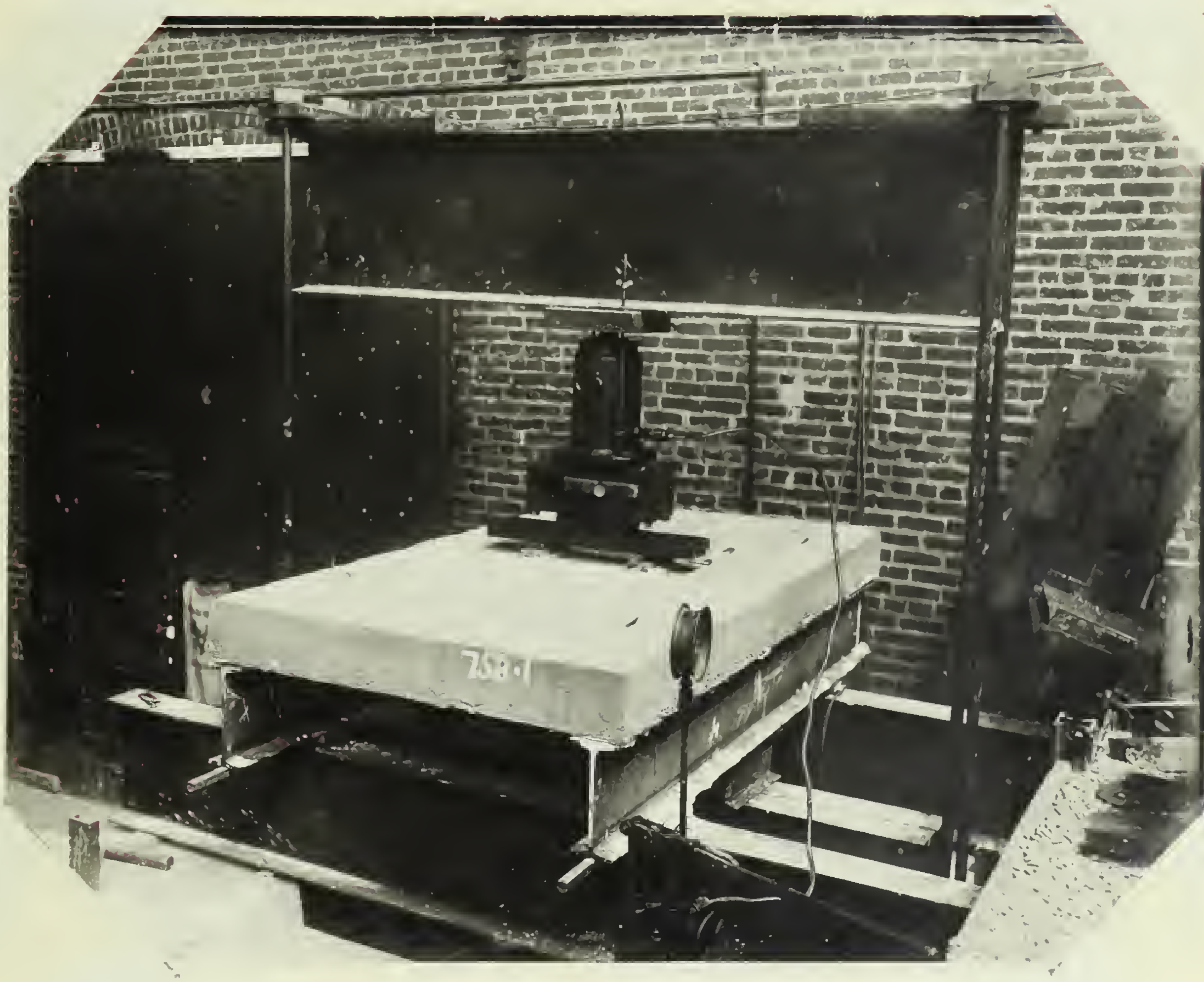
Rod	Total len.
A	53 in
B	57 "
C	49 "
D	41 "
E	55 "
F	43 "
G	35 "

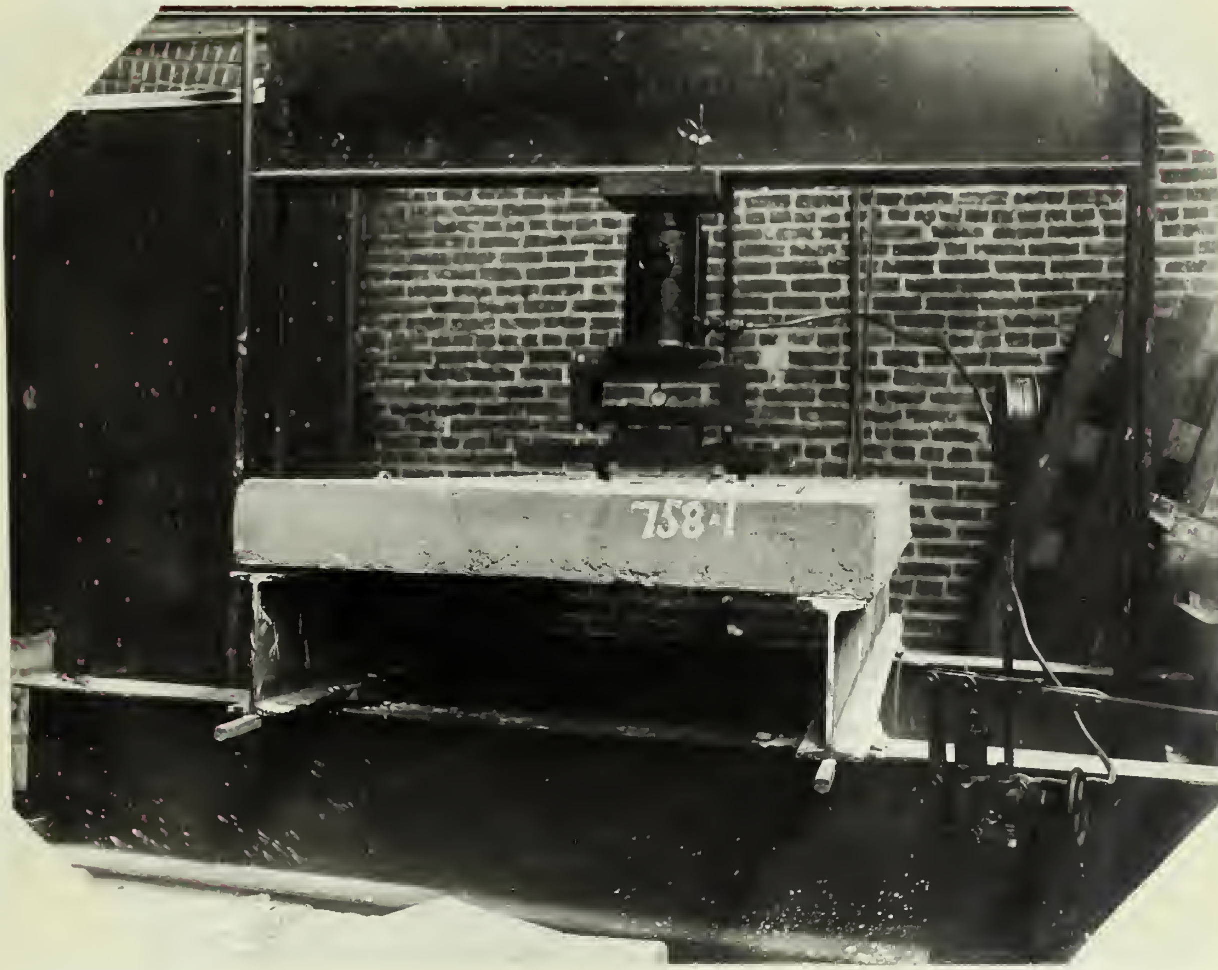
at an age of about 60 days, the exact ages being given in the summary sheet.

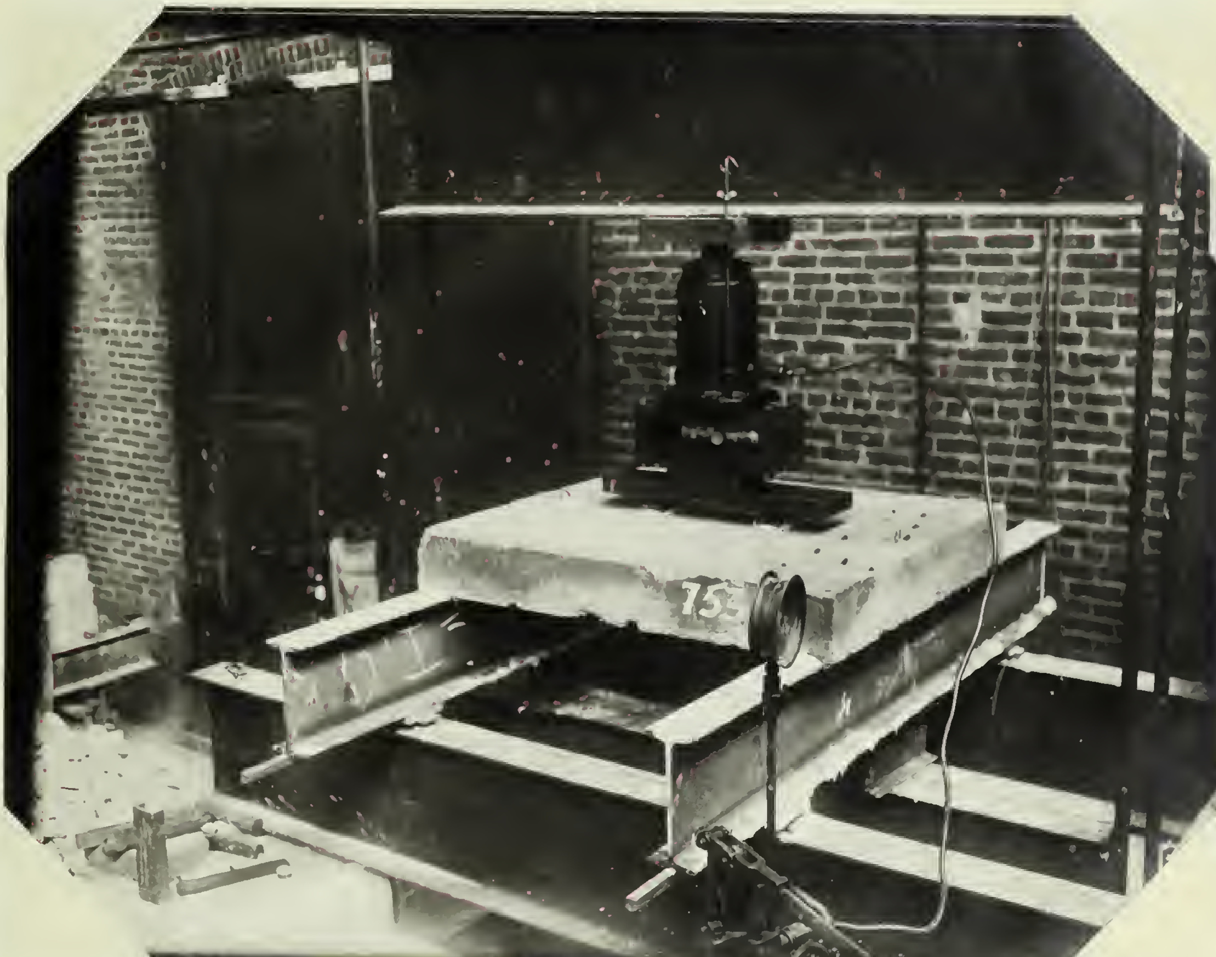
8. Method of Testing.- The beams were tested in a machine designed by the Experiment Station staff for the purpose of testing floor slabs. It was so designed that readings may be taken underneath the slab with the observer standing in a natural position. The load was applied by means of a hydraulic jack. This jack rested on a dynamometer as shown in the photographs on pages 7 to 11. The dynamometer was tested up to 140,000 pounds in one of the Riehle testing machines, and a calibration curve was prepared having the deflections of the dynamometer plotted as ordinates, and the load on the machine as abscissae. The load for any deflection was taken from this calibration curve which is shown on page 11.

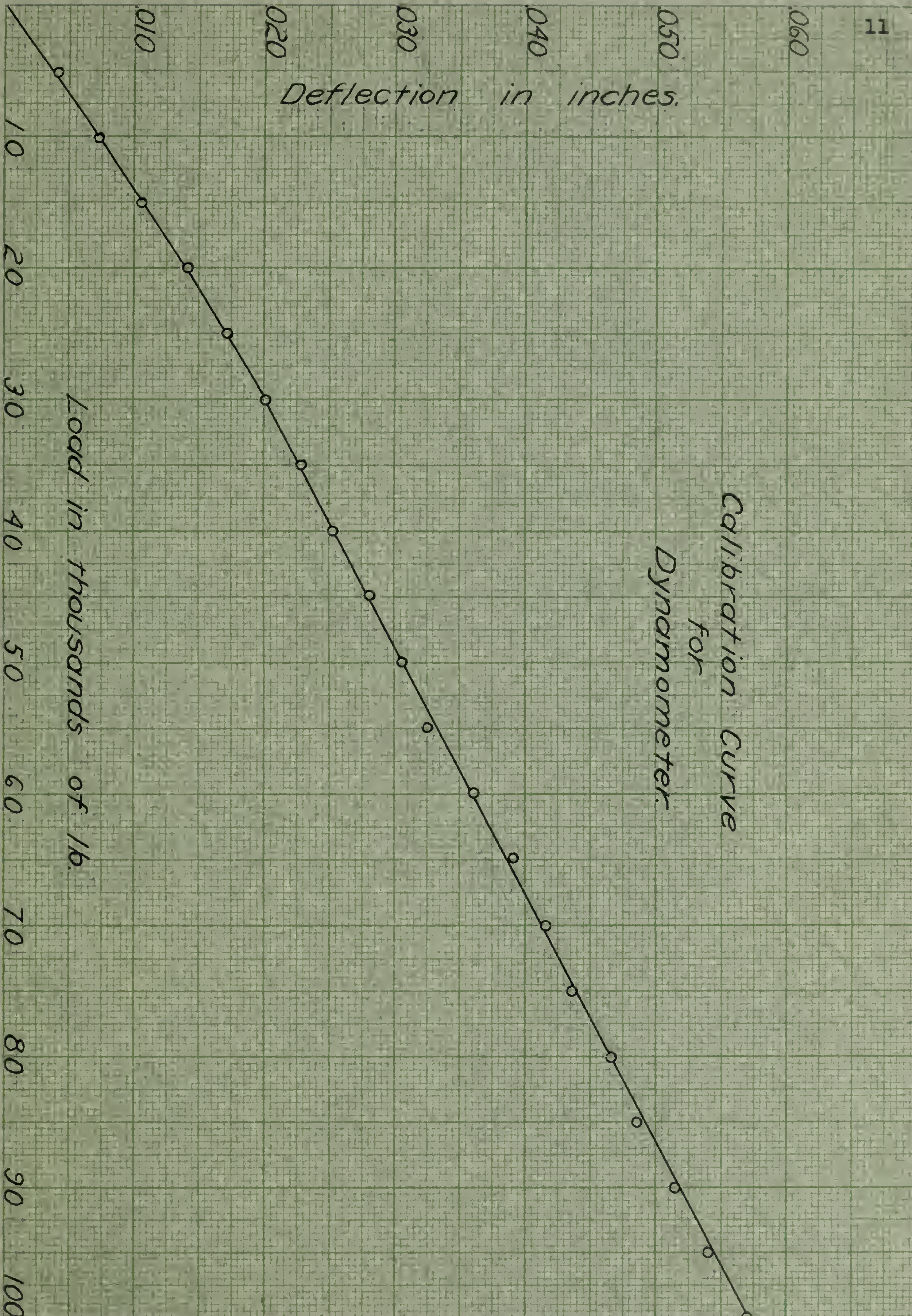
The load blocks were cast-iron bars three inches wide by one inch thick, cut off at different lengths corresponding with the different ratios of loaded width to the total width of the beam. They were imbedded in plaster of paris the same as were the supporting I-Beams on which the beam rested. This was to insure an even bearing on the beam.

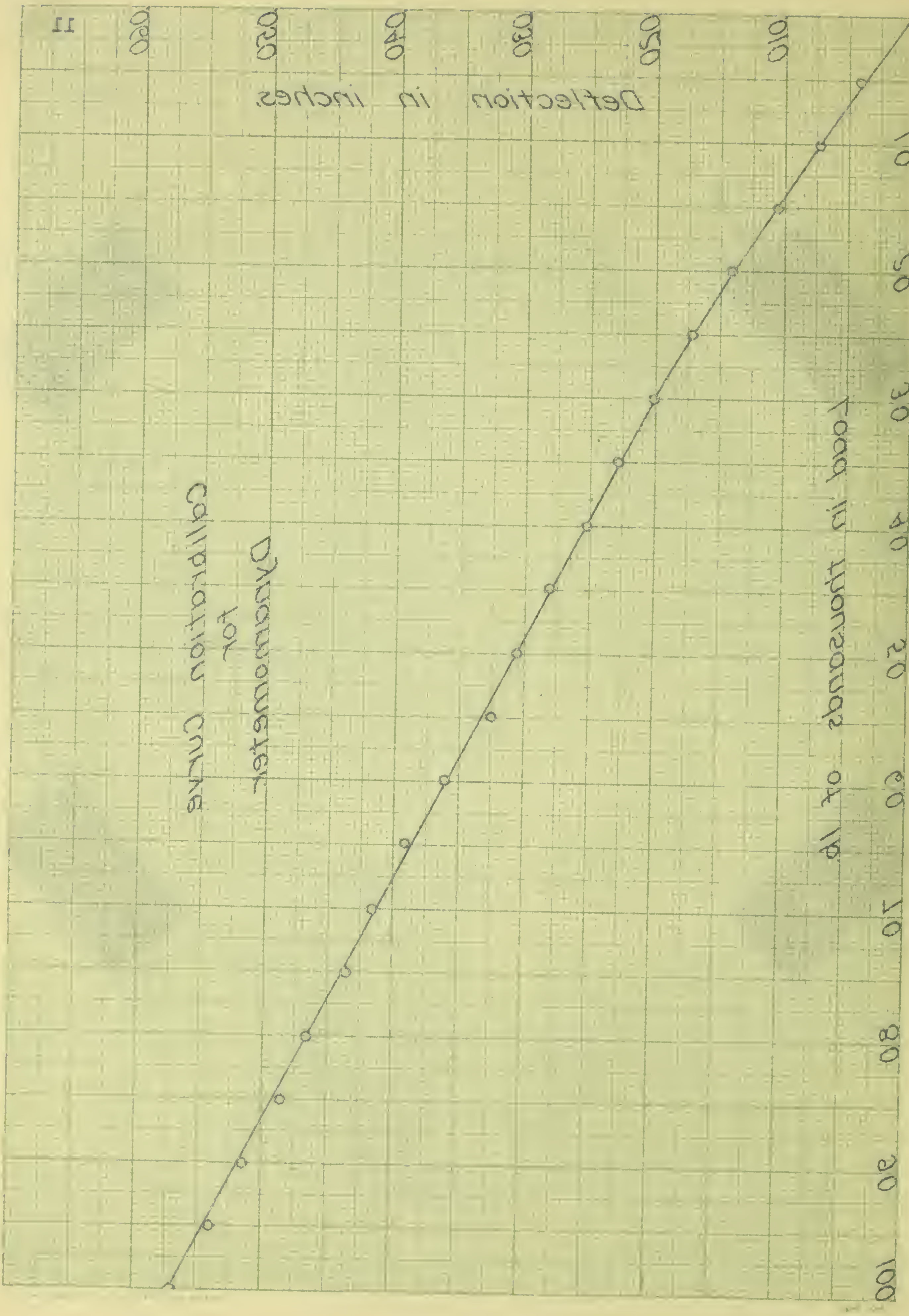
The first beams that were tested were supported by I-Beams as shown in Fig. 1 page 12. This method of support was thought to allow too great a deflection in the 12in. I-Beams, for the wider beams, so the method shown in Fig. 2 , page 12 was adopted. The load was usually applied in about four increments. This necessitated much larger increments than have ordinarily been used in test of reinforced concrete beams, but


758-1


758-1


75


Calibration Curve
for
Dynamometer.
Deflection in inches.
.010
.020
.030
.040
.050
.060
Load in thousands of lb.
10
20
30
40
50
60
70
80
90
100


Calibration Curve
for
Dynamometer
Deflection in inches
Load in thousands of lb
.010
.020
.030
.040
.050
.060
10
20
30
40
50
60
70
80
90
100

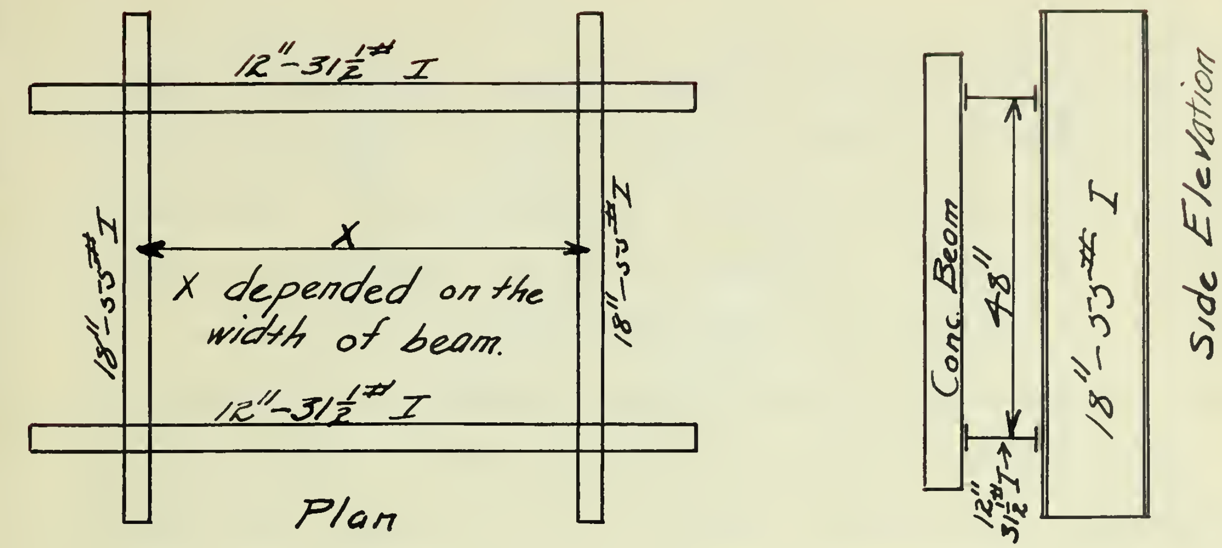
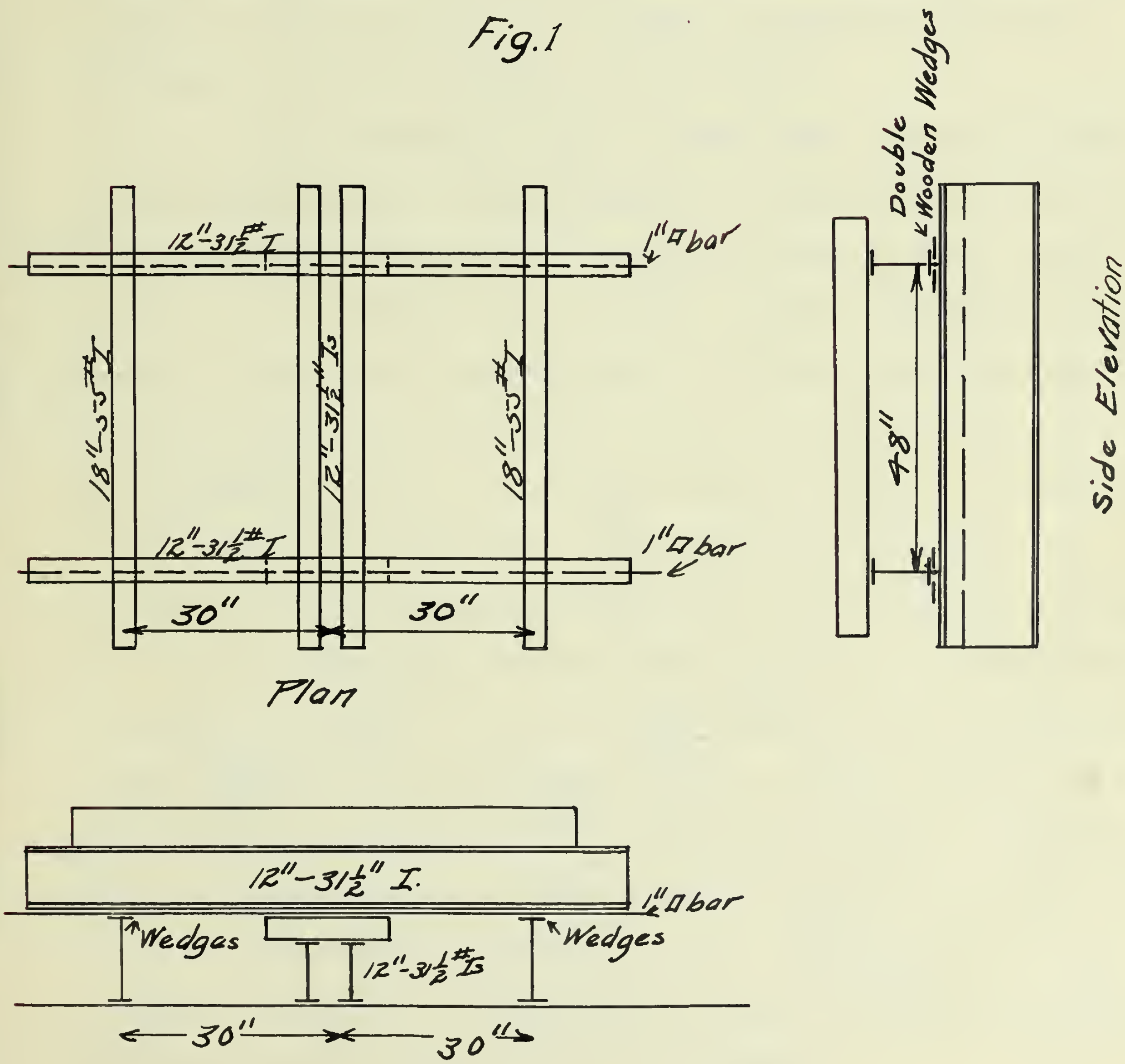


Fig.1

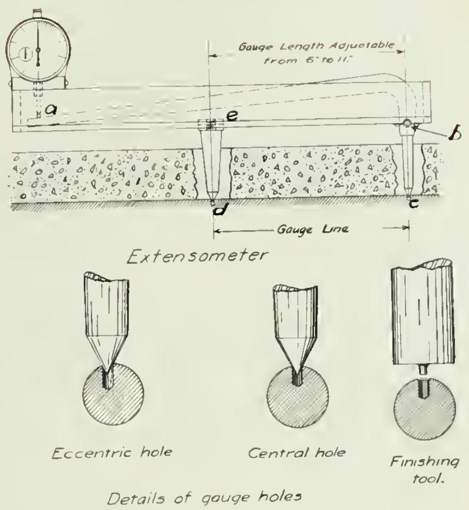


End Elevation

Fig. 2.

owing to the great length of time required to take a complete set of readings, it was necessary to reduce the number of increments. These increments varied, depending on the relative changes in stresses, and the indications of failure. They were smaller near the ultimate load than at the beginning, in order to determine the primary cause of failure, if possible. Since more emphasis was placed on the distribution of stress at a given load, than on the load stress relations the impracticability of applying the load in smaller increments does not seriously affect the value of the results.

The deformations of the steel were measured by the Berry extensometer, the principles of which are shown in Fig. 3 . The arm (a-b) is hinged at (b) and the ratio of the distance (a-b) to the distance (b-c) is five, so that a movement of .001 inches of the dial, corresponds to an actual deformation in the steel of only .0002 inches or a stress of 1000 lb. per. sq. in. if the gauge length is six inches and the modulus of elasticity of the steel is 30,000,000. The leg (d-e) is movable and can be varied from six inch gauge lengths to eleven inch gauge lengths. The amount of movement of the lever arm at (a) was measured by the dial. In order to use this instrument it was necessary to chisel out holes in the concrete to the steel, and then to drill holes in the steel about 3/32 inches deep and 3/64 inches in diameter, and adjust the instruments to these holes which were as nearly six inches apart as possible. The accuracy of the readings depends to a considerable extent on the skill of the operator and the smoothness of the holes.



Gauge Length Adjustable from 6" to 11."
a
e
b
d
c
Gauge Line
Extensometer
Eccentric hole
Central hole
Finishing tool.
Details of gauge holes

Fig. 3

No	Width (Inches)	Reinforcement						Date						Age at Test Days	Measured Stress in Steel					Calculated Stress in Long Steel
		Longitud.			Transv.			Made			Tested				Longitudinal			Transv.		
		%	No. Rods	Dia. Rods	%	No. Rods	Dia. Rods	Mo.	Da.	Yr.	Mo.	Da.	Yr.		Edge lb sq. in	Center lb sq. in	AV. lb sq. in	End lb sq. in	Cent. lb sq. in	
750.1	24	1.07	7	3/8	0	0	0	11	4	11	1	6	12	63	42,000*	42000*	41500*	—	—	57600
751.1	"	"	14	"	"	"	"	"	28	"	2	9	"	73	34,500	32000	33200	—	—	38200
752.1	"	"	"	"	"	"	"	"	"	"	"	6	"	70	29,400	33800	31600	—	—	30200
.2	"	1.56	15	7/16	"	"	"	12	2	"	"	10	"	70	25000	22000	23500	—	—	27100
753.1	48	1.51	29	"	"	"	"	11	10	"	1	12	"	63	10000	20000	15000	—	—	15750
.2	"	"	"	"	"	"	"	"	22	"	2	2	"	72	16000	20000	18000	—	—	15700
754.1	"	"	"	"	.23	4	3/8	"	10	"	1	13	"	64	13000	26000	19500	15000	39000	18500
.2	"	"	"	"	"	"	"	"	25	"	2	6	"	74	16000	36000	26000	28000	42000	21000
755.1	"	"	"	"	.46	8	"	10	26	"	1	5	"	71	22000	40000	31000	42000*	42000*	24800
.2	"	"	22	1/2	"	"	"	12	6	"	2	17	"	73	21000	26000	23500	42000*	42000*	21600
756.1	72	1.50	33	"	.23	4	"	11	1	"	1	2	"	62	10000	35000	22500	42000	42000*	16900
.2	"	"	"	"	"	"	"	12	7	"	2	23	"	78	7000	27000	17000	42000*	42000*	13300
757.1	"	"	"	"	.78	10	7/16	11	18	"	1	19	"	62	14000	28000	21000	42000*	42000*	12000
.2	"	"	"	"	.75	13	3/8	12	9	"	2	24	"	77	10000	23000	16500	30000	42000*	21300
758.1	96	"	44	"	.23	4	"	11	22	"	"	2	"	72	4000	20000	12000	42000*	42000*	12000
.2	"	"	"	"	"	"	"	"	27	"	"	7	"	72	1000	42000*	21500	42000*	42000*	11100
759.1	36	1.02	20	3/8	.78	10	7/16	"	1	"	1	6	"	66	40000	40000	40000	14000	42000	41600
.2	"	"	"	"	.75	13	3/8	"	28	"	2	9	"	73	42000*	42000*	42000*	—	—	35200
760.1	"	"	"	"	.46	8	"	"	4	"	1	8	"	65	42000*	42000*	42000*	18000	22000	42700
.2	"	"	"	"	"	"	"	12	5	"	2	16	"	73	42000	42000	42000	—	—	42000
761.1	"	"	"	"	.23	4	"	11	22	"	1	20	"	59	30000	30000	30000	23000	32000	27700
.2	"	"	"	"	"	"	"	12	5	"	2	23	"	80	41000	38000	39000	—	—	37400
762.1	"	"	"	"	0	0	0	11	25	"	"	3	"	70	24000	26000	25000	—	—	27600
.2	"	"	"	"	"	"	"	12	5	"	"	17	"	74	34000	30000	32000	—	—	25600
763.1	48	1.04	"	7/16	"	"	"	10	26	"	1	2	"	68	12000	42000	27000	—	—	16680
.2	"	1.09	16	1/2	"	"	"	12	6	"	2	17	"	73	17000	20000	18500	—	—	21200
764.1	"	1.04	20	7/16	.78	10	7/16	11	18	"	1	15	"	58	35000	24000	29500	36000	34000	34700
.2	"	"	"	"	.75	13	3/8	12	2	"	2	10	"	70	37000	34000	35500	20000	40000	34400
765.1	72	"	23	1/2	.0	0	0	"	9	"	"	26	"	79	7000	19000	13000	—	—	11400
.2	"	"	"	"	"	"	"	"	5	"	"	17	"	74	8000	20000	14000	—	—	14400
766.1	"	"	"	"	.75	13	3/8	11	29	"	"	10	"	73	18000	40000	29000	28000	42000*	28700
.2	"	"	"	"	"	"	"	12	8	"	"	24	"	78	18000	30000	24000	34000	40000	21800
767.1	96	1.02	39	7/16	.78	10	7/16	11	24	"	"	5	"	73	6000	32000	19000	42000	42000*	19200
.2	"	"	30	1/2	.75	13	3/8	12	4	"	"	12	"	70	10000	38000	24000	36000	42000*	19900
768.1	48	.84	22	3/8	0	0	0	10	25	"	12	27	11	63	18000	22000	20000	—	—	27600
.2	"	"	"	"	.23	4	3/8	"	"	"	1	1	12	68	24000	34000	29000	26000	42000*	30200

Strength of Cubes lb. per sq. in	Bearing lb. sq. in $\frac{W}{0.6b}$	Bond. lb. sq. in $\frac{\text{Max. Meas. S}}{\text{Bond A.}}$	Vertical Shear $v = \frac{V}{bjd}$	Efficiency $\frac{WQ}{P_1} = \frac{f}{f_1}$ 742.1	750.1	Manner of Failure	Load = gauge + 2000 lbs. for app.	No. of beam
3120	970	148	116	1.29	1.00	T_1	14000	750.1
2300	2530	234	153	.85	.66	S & B_1	37000	751.1
2300	1440*	230	177	.67	.52	S & B_1	41500	752.1
2470	2590	128	159	.61	.47	S	37300	.2
2035	1470	116	89	.35	.27	S	42500	753.1
2320	1460	116	88	.35	.27	S	42000	.2
1625	1740	157	105	.41	.32	S	50000	754.1
2610	1980	210	120	.48	.37	S	57000	.2
2310	2330	277	141	.55	.43	S & B_1	67000	755.1
1825	2030	132	122	.48	.37	B_t	58300	.2
2290	1580	178	94	.37	.29	S	68000	756.1
1995	1240	137	74	.30	.23	S	53500	.2
	1770	143	106	.40	.32	B_1	76500	757.1
2530	1900	117	114	.45	.35	B_t & B_1	82000	.2
2130	1110	102	67	.27	.21	S	64000	758.1
2310	1030	214	63	.24	.19	S	59500	.2
2290	2790	272	159	.93	.72	B_1	58000	759.1
2300	2270	286	135	.79	.61	B_1 & D	49000	.2
3120	2760	286	163	.95	.74	B_1	53500	760.1
2275	2710	286	161	.94	.73	B_1 & D.	58500	.2
2320	1790	204	106	.62	.48	B_1 & B_t	38600	761.1
2275	2160	279	128	.75	.58	B_1	46500	.2
2610	1780	177	106	.62	.48	S	38500	762.1
2275	1650	232	98	.44	.34	S	35600	.2
2310	1100	291	65	.37	.29	S	31600	763.1
1825	1430	101	83	.48	.37	S	40300	.2
	2300	204	136	.77	.60	B_1 & B_t	66000	764.1
2470	2280	216	135	.77	.60	B_1 & D	65500	.2
1670	740	97	44	.26	.20	S	32000	765.1
1970	930	101	56	.32	.25	S	40500	.2
2795	1930	202	111	.65	.50	B_1	80700	766.1
1860	1430	152	85	.49	.38	B_1	61500	.2
2475	1240	186	74	.43	.33	B_1	71500	767.1
2740	1280	221	76	.44	.34	B_1	73800	.2
2430	1490	178	87	.62	.48	S	43000	768.1
2430	1630	275	95	.67	.52	S	47000	.2

III EXPERIMENTAL DATA AND DISCUSSION

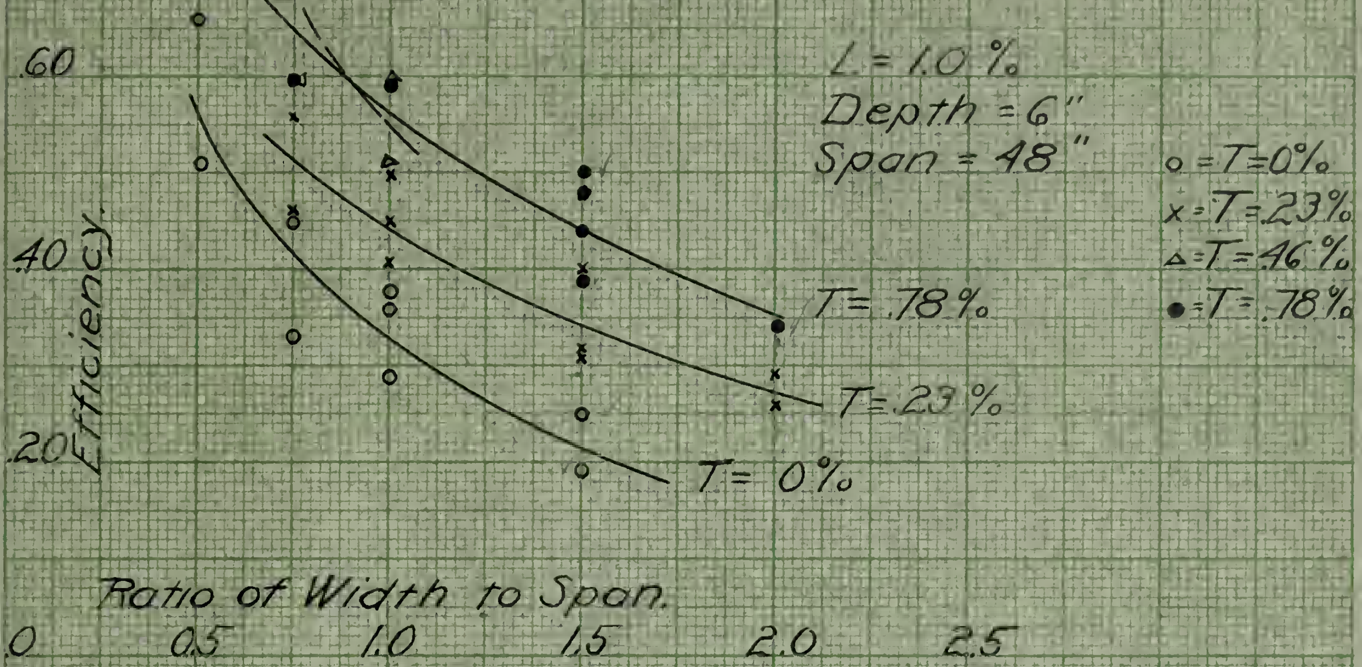
The original data obtained in these tests were recorded in notebooks kept by the Experiment Station and are too voluminous to be copied in this thesis. The results of individual tests are given in the form of a summary, in tables, and in curves plotted from reduced data.

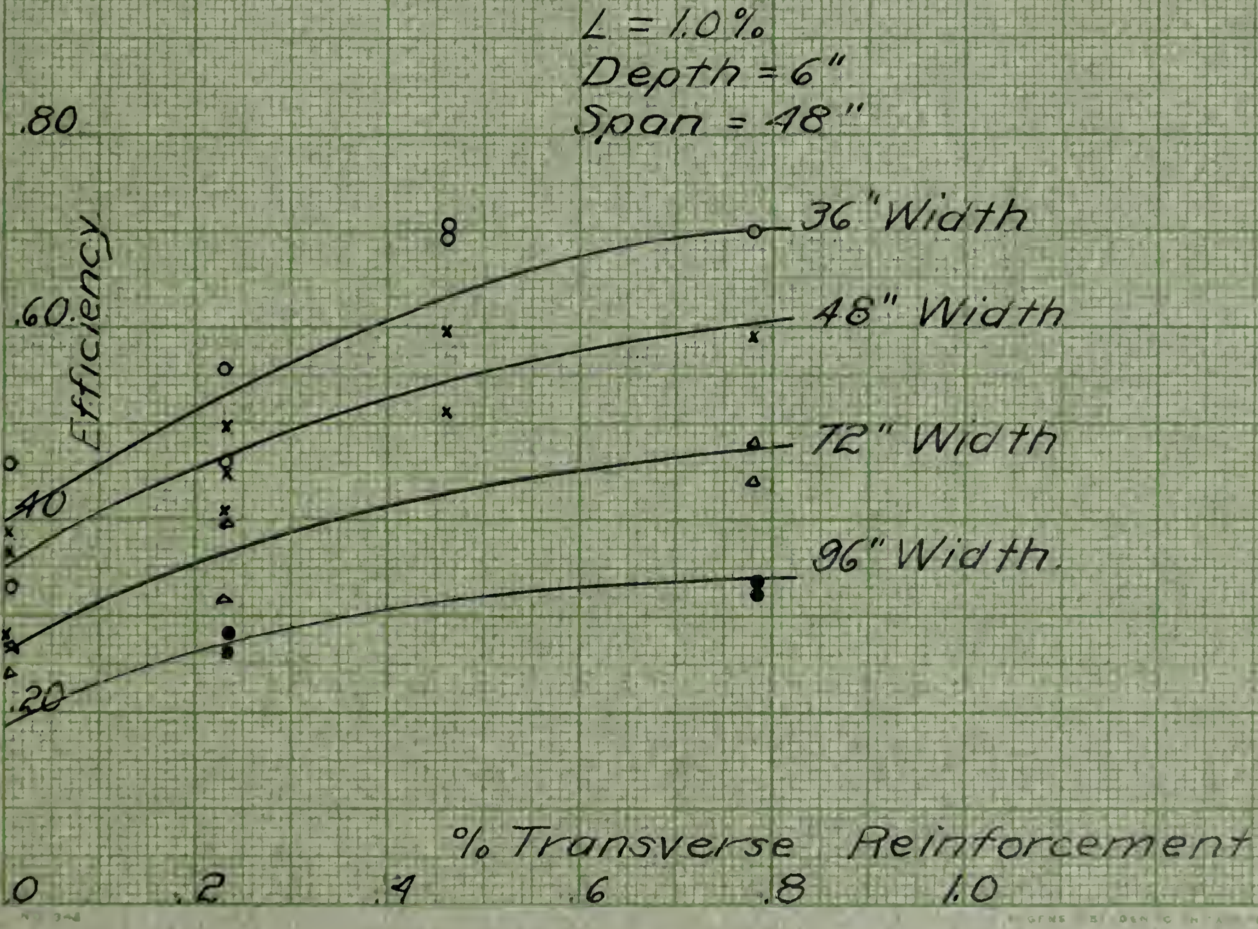
The summary sheets on pages 16 and 17 give the ultimate loads, the manner of failure, the efficiencies as compared with the standard beam, the vertical shear, (v) in lb. per sq. in. the bond stress, the bearing of the load blocks on the beam, the calculated stress and the measured stress.

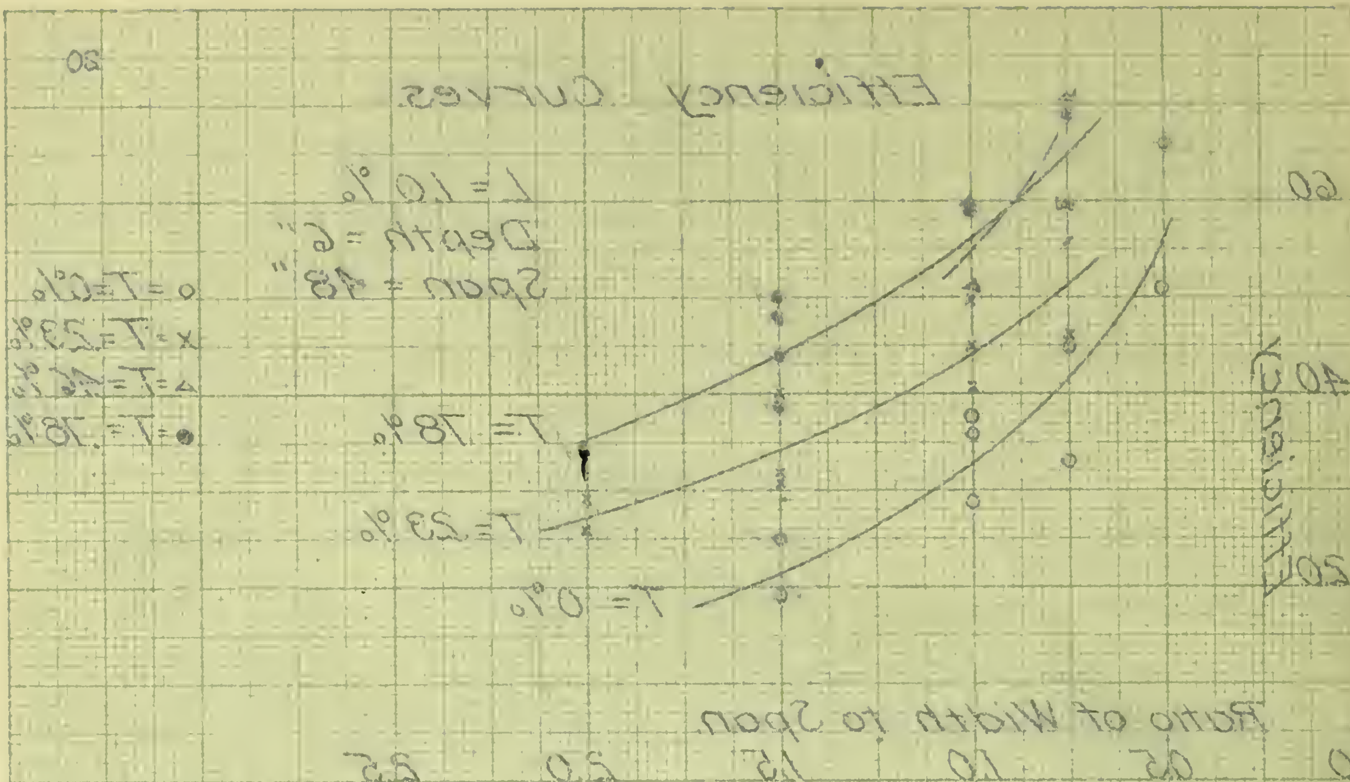
The stresses in the summary sheet that are marked with an asterisk (*) are to indicate that the steel went beyond the elastic limit, and that the true stresses were probably greater than these, but were taken as 42,000 lb. sq. in. as this was about the elastic limit of the steel.

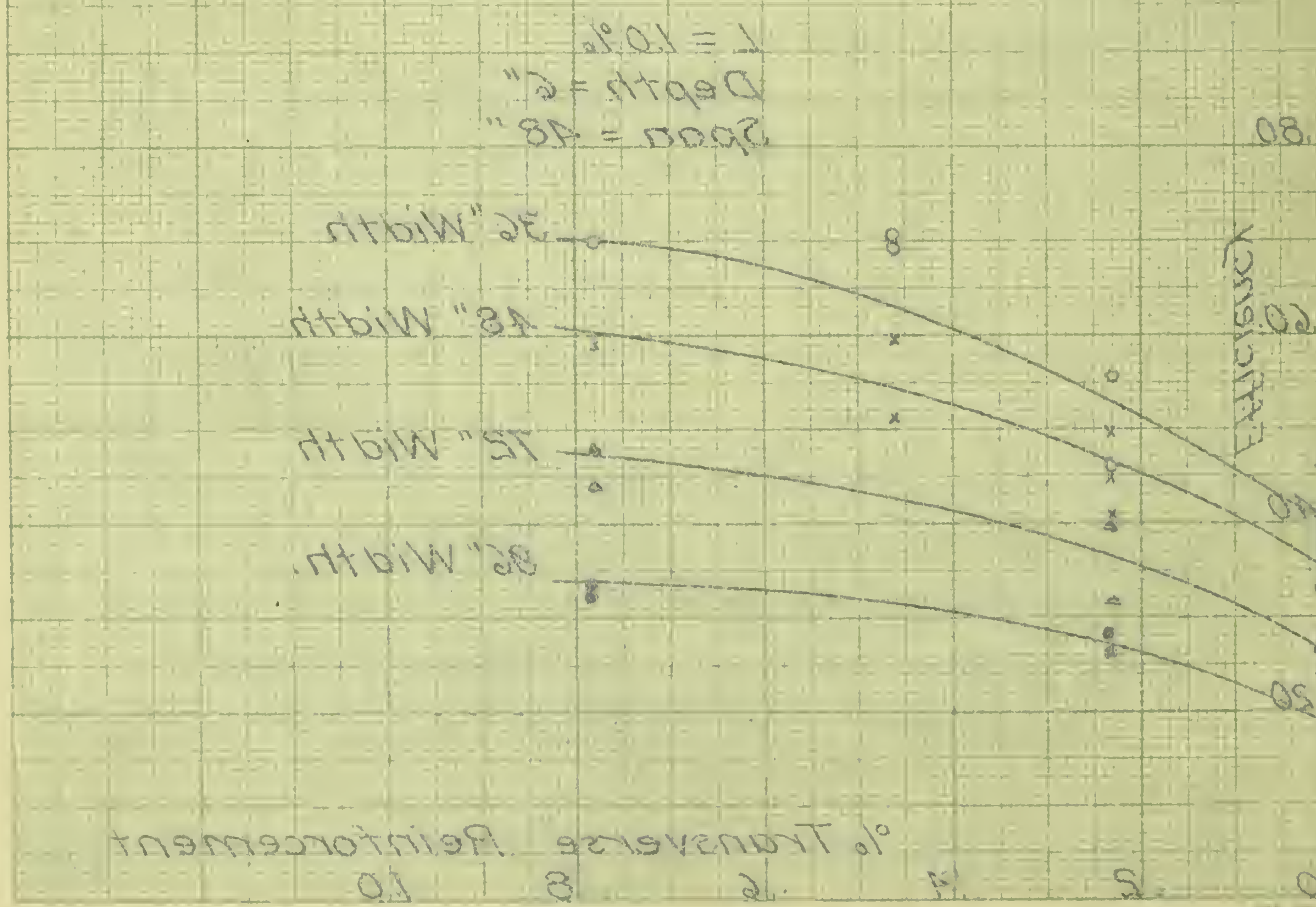
10. Efficiency.- The efficiency of a beam has been defined as the ratio of the load carried, to the load it ought to carry in order to develop the full strength of all the longitudinal steel. The efficiencies of the beams in this series are seen to be low, but this is partly explained by the fact that the standard beam seems to have been exceptionally strong in comparison with the standard of 1911, and judging from the high calculated stress that is shown. Just why the calculated stress of the standard beam should differ so much from the measured stress, while all others agree fairly well, can not be explained. Even though the steel was stressed beyond the elastic limit, the deformation was measured and it did not indicate as high a stress as the calculated stress. The equat-

Efficiency Curves.






Efficiency Curves
L = 1.0 %
Depth = 6"
Span = 48"
o = T = 0%
x = T = 23 %
● = T = 78 %
T = 78 %
T = 23 %
T = 0 %
Efficiency
20
40
60
Ratio of Width to Span
0
0.5
1.0
1.5
2.0
2.5


L = 1.0 %
Depth = 6"
Span = 48"
36" Width
48" Width
72" Width
Efficiency
20
40
60
80
% Transverse Reinforcement
0
.2
.4
.6
.8
1.0

ion for the efficiency is derived in E.J.Schell's thesis page 18. The diagrams on page 20 of this thesis show the efficiencies of the various beams. The curves at the top of the same page show how the efficiency varies with the ratio of width to span, the longitudinal reinforcement being reduced to the same basis. The curves at the bottom of the page show the way in which the efficiency varies with the transverse reinforcement.

11. Shearing Stresses.- The vertical shearing stress shown in the summary sheet is the average for any cross section of the beam between the load and the support. It is likely that this is less than the shearing stress between the load and the support on a central longitudinal strip.

12. Bond Stresses.- The bond stresses are obtained by dividing the maximum measured stress in the rods by the bond area of that rod. In a bent up rod the bond area required should be sufficient to resist the stress due to the moment, in addition to the diagonal tensile stress carried by the rods.

13. Bearing Stresses.- The bearing blocks were three inches wide and one-tenth the width of the beam, with the exception of beam No. 752.1, which was two-tenths, making the total bearing area in sq. in. equal to 2x3x.1xb = 0.6b where b is given in inches. The bearing values in many cases are shown to be higher than the compressive strength of cubes of the same quality of concrete. Just what effect these high bearing values had on the manner of failure is not known.

14. Ultimate Loads.- For a given beam the recorded load is 2000 pounds greater than the load indicated by the dynamometer. This is to correct for the weight of the dynamometer,

jack, and the 24 inch I-Beam above the jack.

15. Manner of Failure.- The manner of failure is designated by symbols as follows;

T_l indicates a failure by tension in the longitudinal steel.

S indicates a slab failure. In this thesis any tension failure of either the concrete or steel in the transverse direction, which results in a longitudinal crack in the center, has been designated as a slab failure.

B_l and B_t indicate failures due to slipping of the longitudinal and transverse rods respectively.

D indicates failure by diagonal tension. Tension failures in steel and diagonal tension failures are discussed in bulletins No. 4 and 14 of the Experiment Station.

All the beams of this series failed either as a slab or because of insufficient bond. The only exceptions are beams No. 764.2, 760.2 and 759.2. These failed by bond and diagonal tension. In all cases except beams No. 759.1 and 764.1 the stresses were higher in the transverse steel than in the longitudinal steel at failure. This seems to indicate that these two beams were the only ones over-reinforced in the transverse direction. They failed by bond but had high stresses in the longitudinal steel. Since the experimental determination of the proper amount of transverse reinforcement depends largely on the results of tests in which failure is caused by tension in either the longitudinal or the transverse reinforcement, it is unfortunate that so many of these beams failed in bond.

16. Load Stress Diagram.- The data obtained in regard to stresses are plotted on pages 32to 111. The diagrams having loads as ordinates and stresses as abscissae show the loads and the corresponding stresses for various bars of the beam. Stresses were not measured in all the bars. Only enough bars were measured to show the general distribution of stress across the beams.

The diagrams having stresses as ordinates and width of beam as abscissae show the general distribution of stress. Two loads were chosen at which the stresses in the longitudinal steel were plotted and three loads for stresses in the transverse steel. In each case the stresses were plotted for such a load as stressed the steel, first, to about 15,000 lb. per. sq. in., and second, to about 40,000 lb. per. sq. in. The third load for the transverse steel was that which stressed the longitudinal steel to about 15,000 lb. per. sq. in. These curves show that the narrower the beam the less the amount of transverse reinforcement required. They indicate also, that the stress in the transverse steel increases at a more rapid rate than does the load, whereas this is not true of the stresses in the longitudinal steel.

17. Deflection Diagrams.- The diagrams from page 112 to116show the curling of the beams in the transverse direction, and the deflection of the supporting I-Beams. The increased deflection of the supporting I-Beams at the maximum load indicate that at failure the load is all concentrated in the central portion of the beam. An exact mathematical solution, showing why the beams curl up from the supports near the outer edges, seems to be an impossibility, but the following considerations

may throw some light on the subject. The radius of curvature of a beam, having a constant moment of inertia varies inversely as the bending moment, and as the bending moment in the transverse direction is greatest at the center due to the load being concentrated there, the radius of curvature is the least at that point. The stiffness of concrete tends to keep all sections in the same position with respect to each other, consequently the tendency due to this sharp radius of curvature at the center is to lift the ends of the transverse sections from the supports, thus forming cantilever beams of the outer transverse sections.

18. Theory:- The fact that wide beams fail by tension in a transverse direction is the evidence that there must be transverse flexure, since there are no forces applied transversely. The only way in which load can be carried to the member when considered as a transverse beam, is through shear between adjacent longitudinal strips.

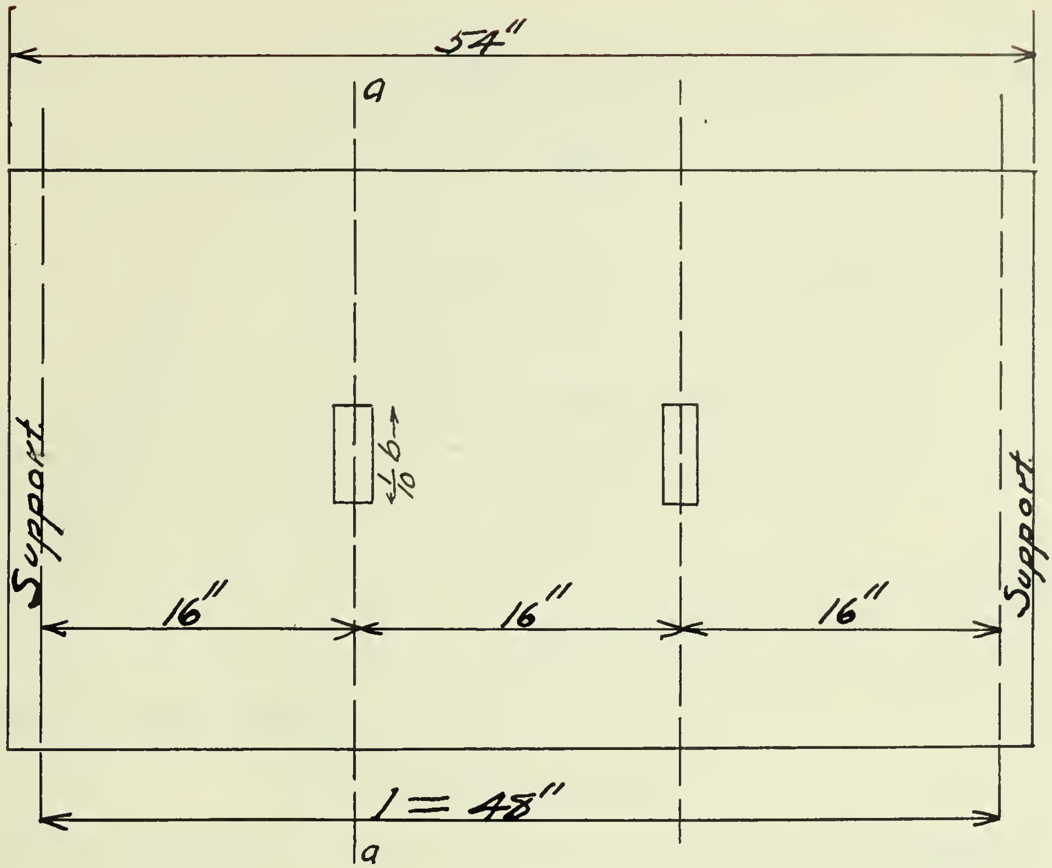
The data of the tests show that the stresses in the longitudinal steel fall off gradually, from a maximum at the center to practically zero at the edge for a beam 96 inches wide, and that the amount of transverse reinforcement required varies with varying ratios of widths to length, increasing as the ratio of width to length increases. In order to determine the proper ratio of transverse reinforcement to the longitudinal reinforcement, the following equation has been derived, based on the assumption, (1), that the shear between any two adjacent strips is approximately proportional to the difference between the steel stresses in the two strips, (2), that for the same condition of loading, the load carried by any longitudinal strip of a beam, is proportional to the stresses developed in the longitudinal steel in that strip, and (3), that the load is not distributed over a width greater than twice the span.

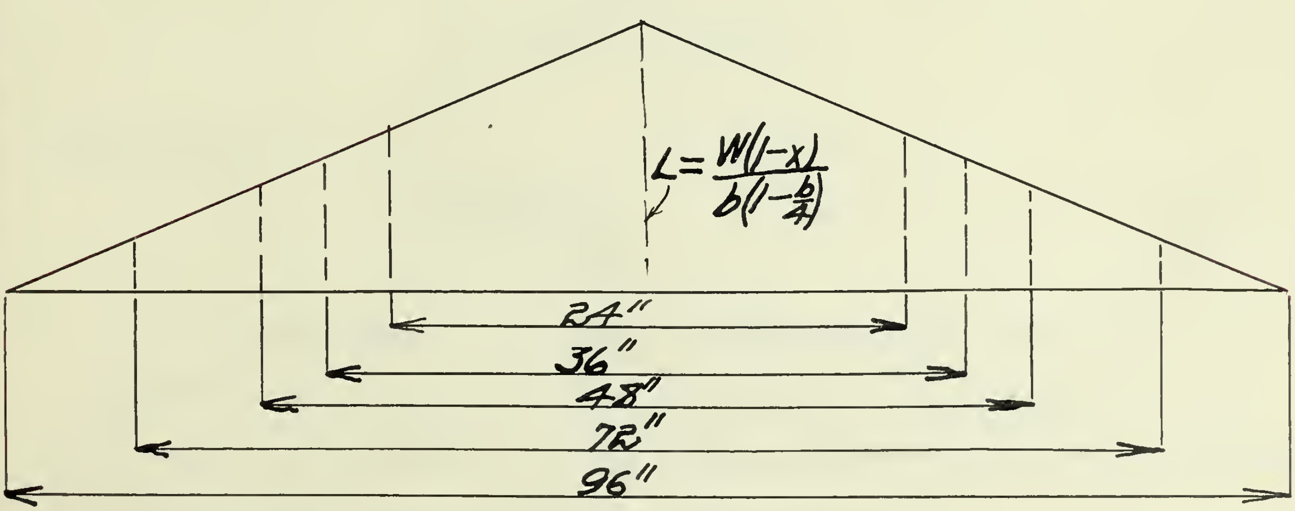
According to the above assumptions, the load curve is a straight line, as shown on page 26.

Let $l =$ span.

$K =$ intensity of load at center.

$x =$ any distance out from the center.





Load Curve. Sec. through (a-a)

L = intensity of load at distance x.

W = load on beam.

b = width.

d = effective depth of longitudinal sections.

f = stress in longitudinal steel.

A = area of longitudinal steel.

jd = arm of resisting couple.

M = bending moment.

p = per. cent of longitudinal reinforcement.

M_t, $A_t j_t d_t$, f_t, d_t, and p_t corresponding values in the transverse sections.

V_t = transverse shear.

Then the intensity of the load at any point at a distance x is

$$L=\frac{K(l-x)}{l}$$

When $x=\frac{b}{2}$ or 1/2 the width of the beam the area under the curve must equal the total load W or

$$\left[\frac{K+K\left(\frac{l-\frac{b}{2}}{l}\right)}{2}\right]\frac{b}{2}=\frac{W}{2}$$

Solving for K

$$K=\frac{Wl}{b\left(l-\frac{b}{4}\right)}$$

This means that under the assumptions used for load distribution the stresses produced at the center due to a load "W" would be equivalent to the stress produced by a load uniformly distributed over the entire width equal to

$$\left[\frac{Wl}{b\left(l-\frac{b}{4}\right)}\right] x\ b = \frac{Wl}{\left(l-\frac{b}{4}\right)}$$

$$L = \frac{W(l-x)}{b\left(l-\frac{b}{4}\right)}$$

$$V_t = \int L dx = \int\left[\frac{W(l-x)}{b\left(l-\frac{b}{4}\right)}\right] dx = \frac{W}{b\left(l-\frac{b}{4}\right)}\left(lx - \frac{x^2}{2}\right) + C$$

Determining the constant of integration by the condition that $V_t = 0$ when $x = \frac{b}{2}$.

$$V_t = \frac{W}{b\left(l-\frac{b}{4}\right)}\left(lx - \frac{x^2}{2}\right) - \frac{W}{2}$$

$$\int V_t dx = M_t = \int\left[\frac{W}{b\left(l-\frac{b}{4}\right)}\left(lx - \frac{x^2}{2}\right) - \frac{W}{2}\right] dx = \frac{W}{b\left(l-\frac{b}{4}\right)}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) - \frac{Wx}{2} + C_1$$

$$M_t = 0 \text{ when } x = \frac{b}{2}$$

$$M_t = \frac{W}{b\left(l-\frac{b}{4}\right)}\left(\frac{lx^2}{2} - \frac{x^3}{6}\right) - \frac{Wx}{2} + \frac{Wb}{4} - \frac{Wb}{\left(l-\frac{b}{4}\right)}\left(\frac{l}{8} - \frac{b}{48}\right)$$

M_t is a maximum when $x = 0$

$$\text{Max. } M_t = \frac{Wb}{8\left(l-\frac{b}{4}\right)}\left(l - \frac{b}{3}\right)$$

Also $M_t = f_t A_t j_t d_t$

For economical design f_t at the center should equal f for a load W.

The data of these tests seem to indicate as a fair

assumption that "f" at the center due to a concentrated load "W" is the same as would be caused by a load of intensity equal to $\dfrac{Wl}{b\left(1-\frac{b}{4}\right)}$ (as derived on page 28), distributed across the full width of the beam at the one-third points of the span.

This would equal a total load of $\dfrac{Wl}{b\left(1-\frac{b}{4}\right)} \times b$ or $\dfrac{Wl}{\left(1-\frac{b}{4}\right)}$

$$M = K_1 Pl \quad \text{and} \quad P = \frac{Wl}{\left(1-\frac{b}{4}\right)}$$

K_1 coefficient depending on loading.

Substitute for P its value above

$$M = \frac{K_1 Wl^2}{\left(1-\frac{b}{4}\right)}$$

$$f = \frac{M}{Ajd} = \frac{K_1 Wl^2}{\left(1-\frac{b}{4}\right)Ajd}$$

Now $A_t = \dfrac{M_t}{f_t j_t d_t}$

Put $f_t = f$ and substitute value of M_t.

$$\text{Then } A_t = \frac{\dfrac{Wb\left(1-\frac{b}{3}\right)}{8\left(1-\frac{b}{4}\right)}}{\left(\dfrac{K_1 Wl^2}{\left(1-\frac{b}{4}\right)Ajd}\right) j_t d_t} = \frac{Ab\left(1-\frac{b}{3}\right)}{8K_1 l^2} \times \frac{jd}{j_t d_t}$$

j_t = practically j.

$$A_t = \frac{Ab\left(1 - \frac{b}{3}\right)}{8\ K_1 l^2} \quad x \quad \frac{d}{d_t}$$

$$A_t = p_t l d_t \qquad A = pbd$$

$$p_t l d_t = \frac{pb^2 d\left(1 - \frac{b}{3}\right)}{8\ K_1 l^2} \quad x \quad \frac{d}{d_t}$$

$$\frac{p_t}{p} = \frac{b^2\left(1 - \frac{b}{3}\right)}{8\ K_1 l^2} \quad x \left(\frac{d}{d_t}\right)^2$$

Put $b = K_2 l$

$$\text{Then } \frac{p_t}{p} = \frac{K_2^2\ l^2\left(1 - \frac{K_2 l}{3}\right)}{8\ K_1 l^3} \quad x \quad \left(\frac{d}{d_t}\right)^2$$

$$= \frac{K_2^2\left(1 - 1/3\ K_2\right)}{8\ K_1} \quad x \left(\frac{d}{d_t}\right)^2$$

This formula is stated in general terms, but since all the beams of this series were of the same span and had a minimum ratio of depth to span of 1/16, further test on beams of different spans and depths will be necessary in order to determine its adaptibility to other conditions.

19. Summary.- The following statements are to be taken not as final conclusions, but as a partial interpretation of the results of these tests.

1. The amount of transverse reinforcement required for narrow beams is very small. It seems that for beams having a width of one-half the span, or less, no transverse reinforcement is necessary.

2. The amount of transverse reinforcement required, increases as the width of beam increases.

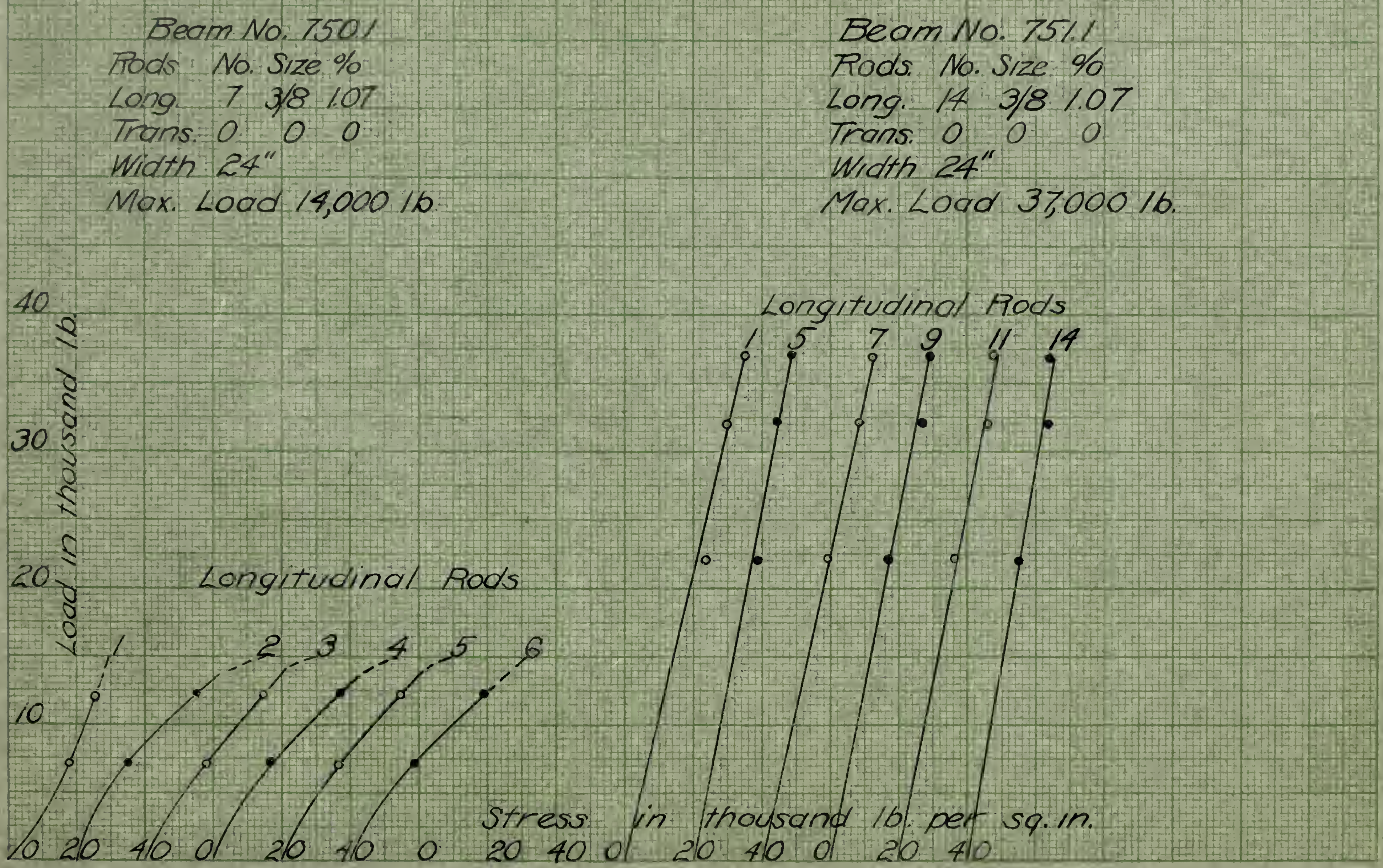
3. The stresses in the longitudinal steel under a concentrated load are greater than for the same load extending over the full width of the beam.

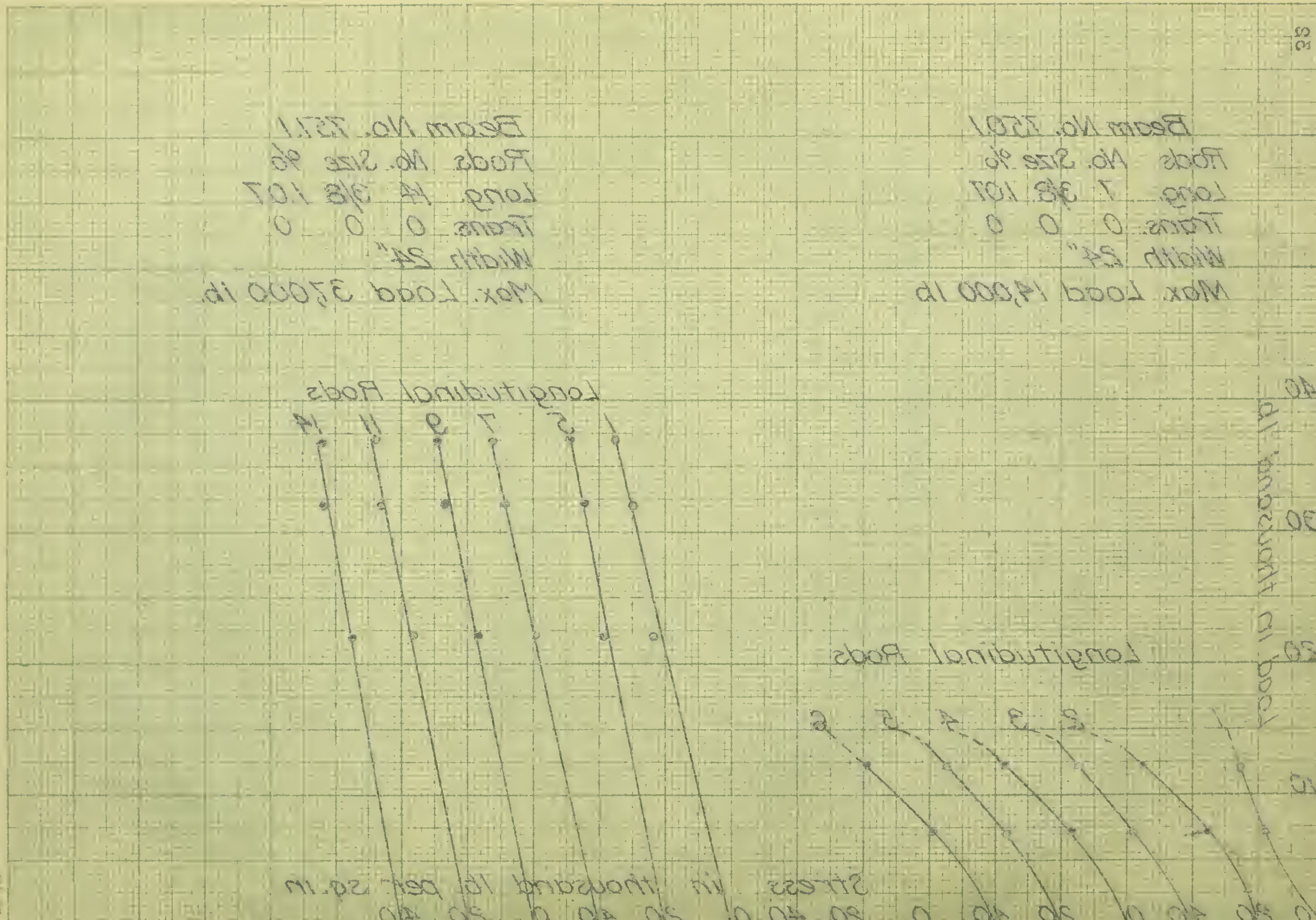
4. The stresses in the longitudinal steel fall off gradually from the center towards the outer edges, reaching zero at a width equal to about twice the span.

5. The stresses in the longitudinal and transverse steel increase at different rates, the transverse stresses increasing more rapidly near the ultimate load.

6. In the continuation of this investigation, it is necessary to make other provisions than were made in the beams of this series to prevent bond failures.

Since the tests are limited in their scope as stated on page 30, any application of these interpretations to other conditions should not be made without considering these limitations.


Beam No. 750.1
Rods No. Size %
Long. 7 3/8 1.07
Trans. 0 0 0
Width 24"
Max. Load 14,000 lb.
Beam No. 751.1
Rods. No. Size %
Long. 14 3/8 1.07
Trans. 0 0 0
Width 24"
Max. Load 37,000 lb.
Load in thousand lb.
40
30
20
10
Longitudinal Rods
1 2 3 4 5 6
Longitudinal Rods
1 5 7 9 11 14
Stress in thousand lb. per sq. in.
0 20 40 0 20 40 0 20 40 0 20 40 0 20 40



Beam No. 7511
Rods No. Size %
Long. 14 3/8 1.07
Trans. 0 0 0
Width 24"
Max. Load 37,000 lb.
Beam No. 1501
Rods No. Size %
Long. 7 3/8 1.07
Trans. 0 0 0
Width 24"
Max. Load 14,000 lb
Longitudinal Rods
1 5 7 9 11 14
Longitudinal Rods
1 2 3 4 5 6
Load in thousand lb
10 20 30 40
Stress in thousand lb per sq. in

Beam No. 750.1

Rods	No.	Size	%
Long.	7	3/8"	1.07
Trans.	0	0	0

Width 24"
Max. Load 14,000 lb.

Beam No. 751.1

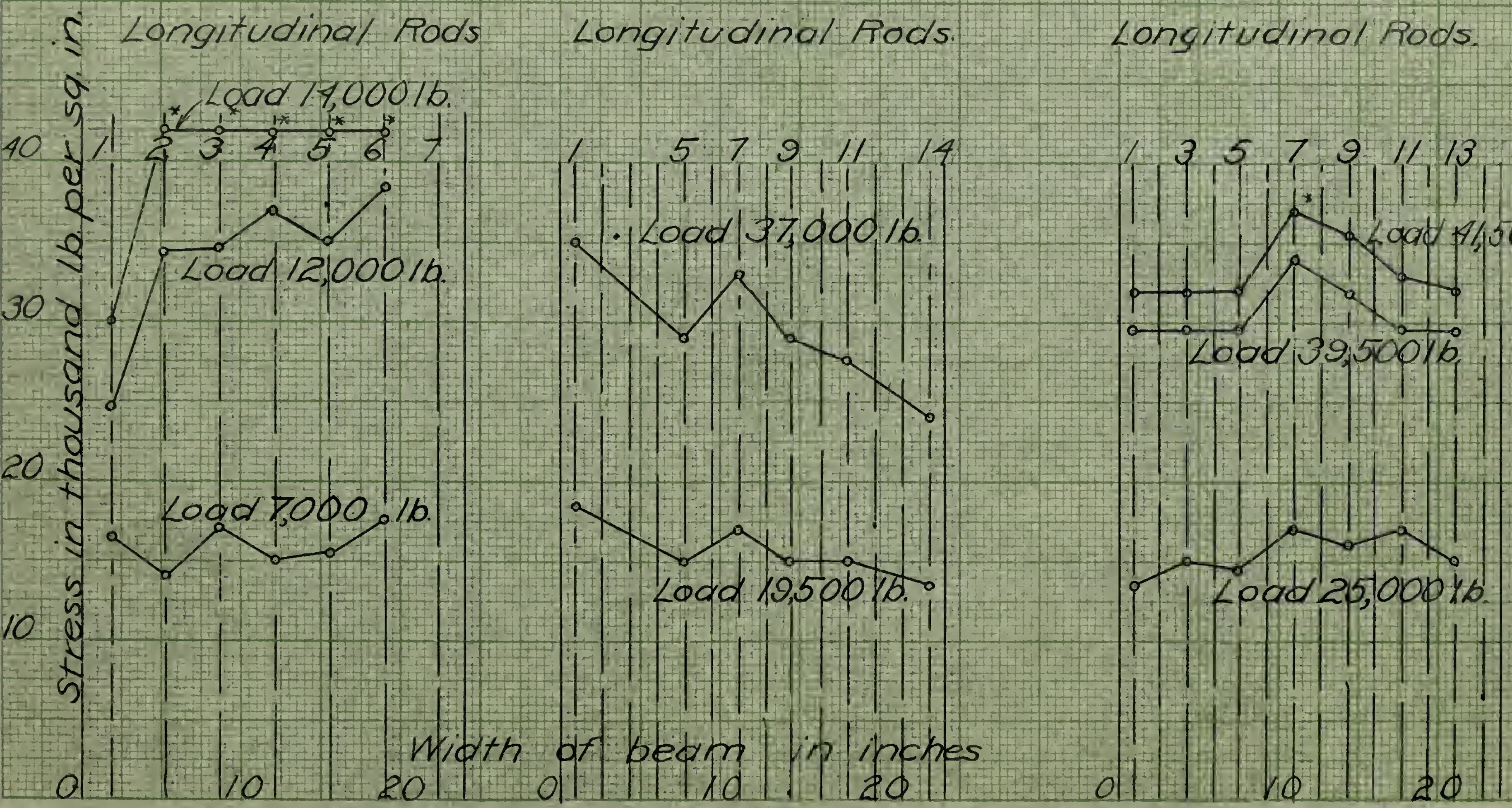
Rods	No.	Size	%
Long.	14	3/8"	1.07
Trans.	0	0	0

Width 24"
Max. Load 37,000 lb.

Beam No. 752.1

Rods	No.	Size	%
Long.	14	3/8"	1.07
Trans.	0	0	0

Width 24"
Max. Load 41,500 lb.



Beam No. 752.1

Rods	No.	Size	%
Long.	14	3/8"	1.07
Trans.	0	0	0

Width 24"

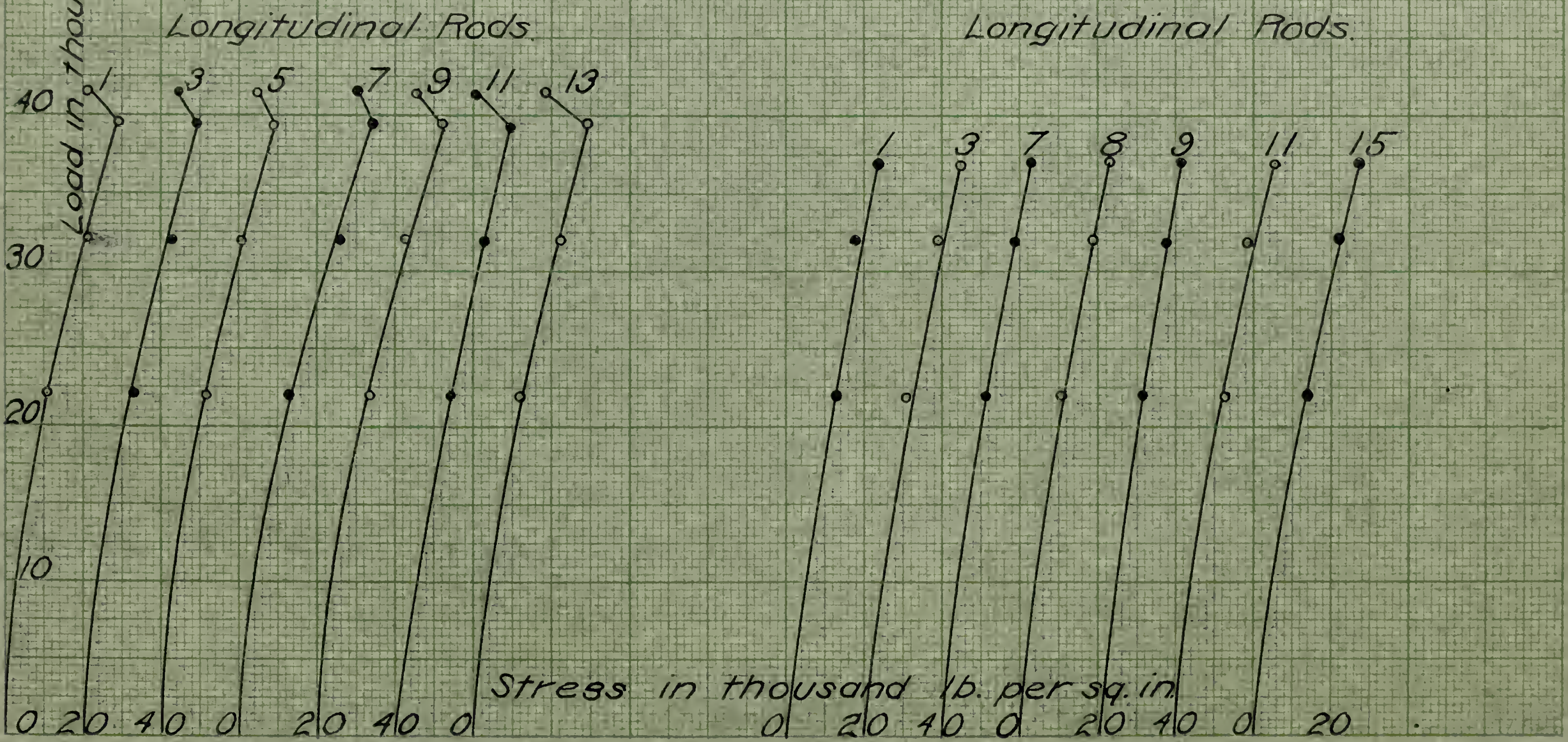
Max. Load 41,500 lb.

Beam No. 752.2

Rods	No.	Size	%
Long.	15	7/16"	1.56
Trans.	0	0	0

Width 24"

Max. Load 37,300 lb.



Beam No. 752.2

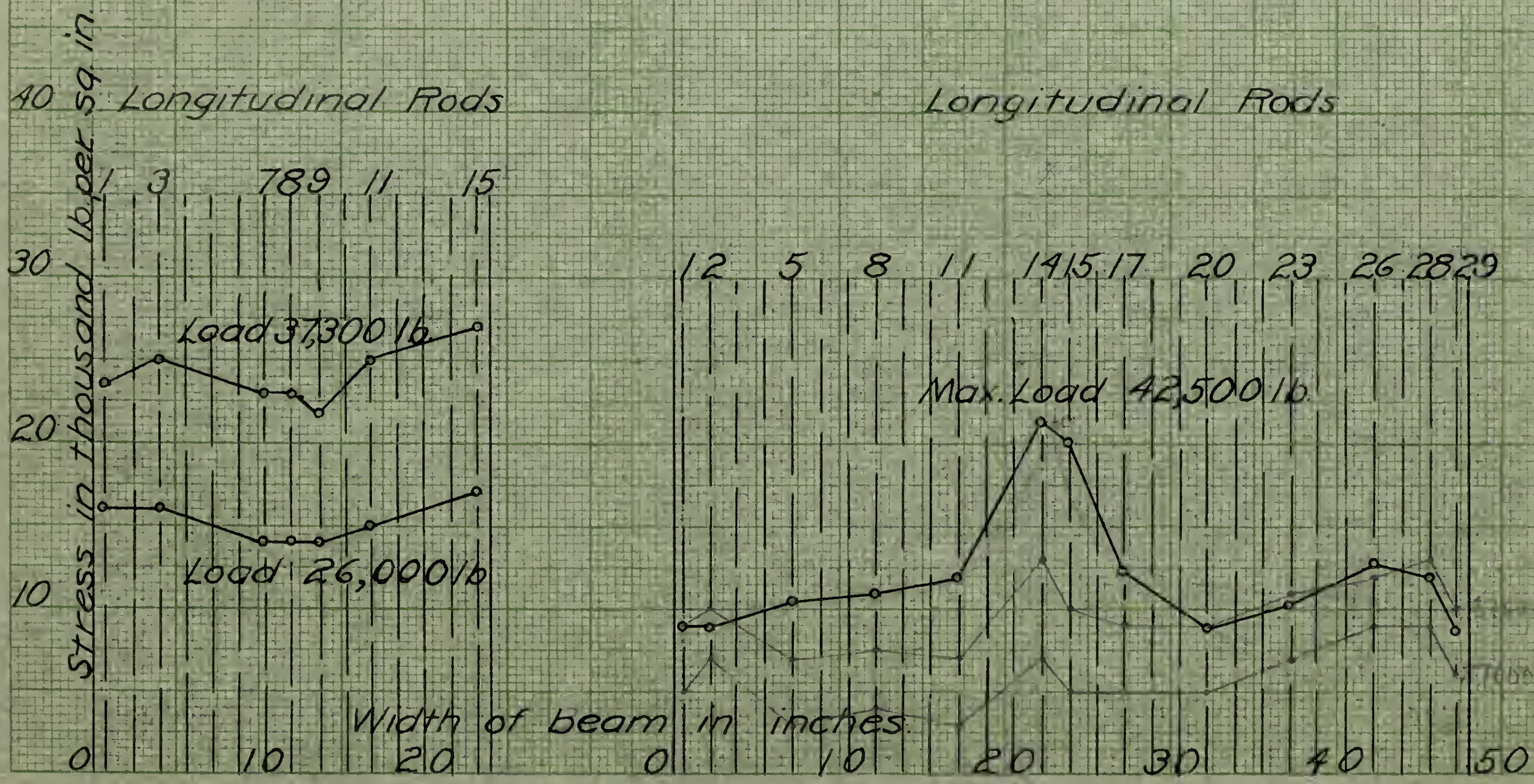
Rods	No.	Size	%
Long.	15	7/16"	1.56
Trans.	0	0	0

Width 24"
Max. Load 37,300 lb.

Beam No. 753.1

Rods	No.	Size	%
Long.	29	7/16"	1.51
Trans.	0	0	0

Width 48"
Max. Load 42,500 lb.

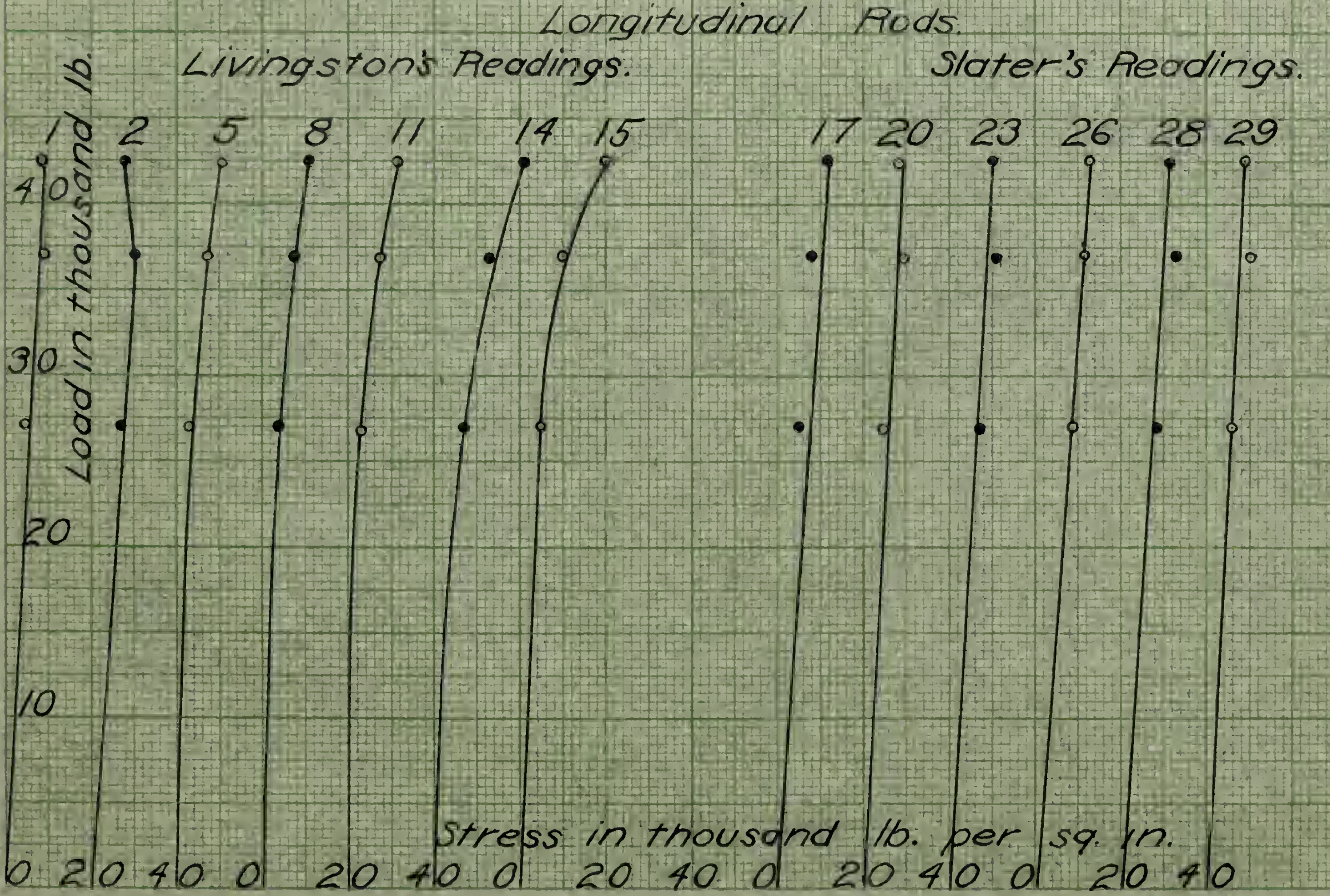


Beam No. 753.1

Rods	No.	Size	%
Long.	29	7/16"	1.51
Trans.	0	0	0

Width 48"

Max. Load 42,500 lb.

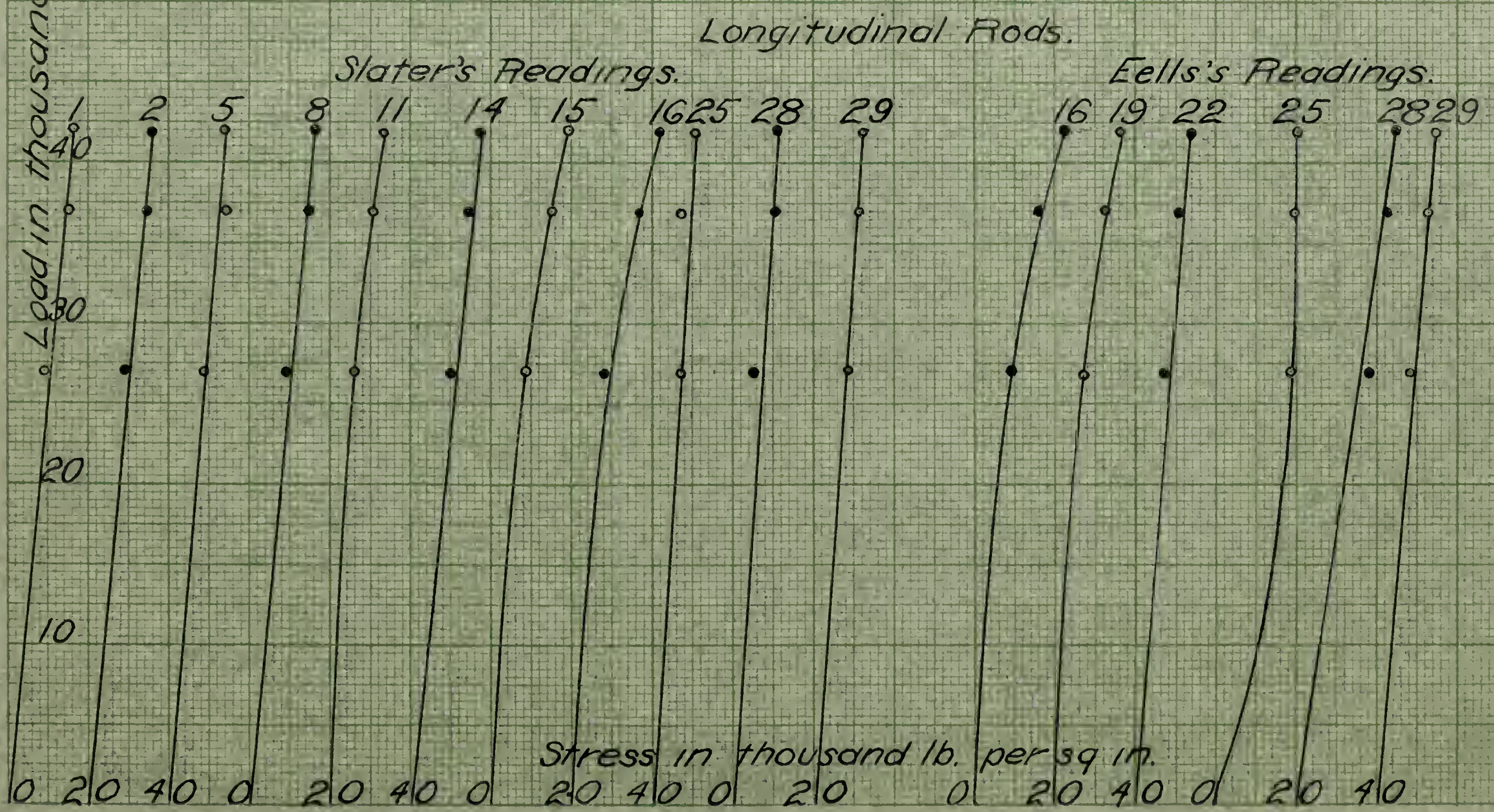


Beam No. 75 3.2

Rods	No.	Size	%
Long.	29	7/16"	1.51
Trans.	0	0	0

Width 48"

Max. Load 42,000 lb.



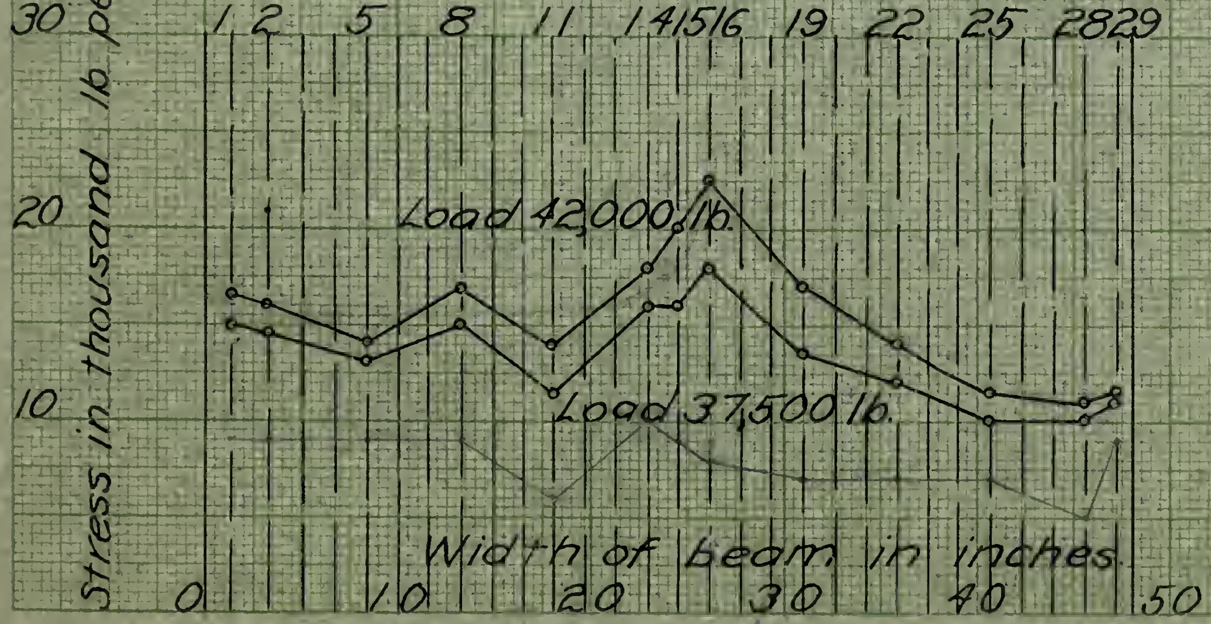
Beam No. 753.2

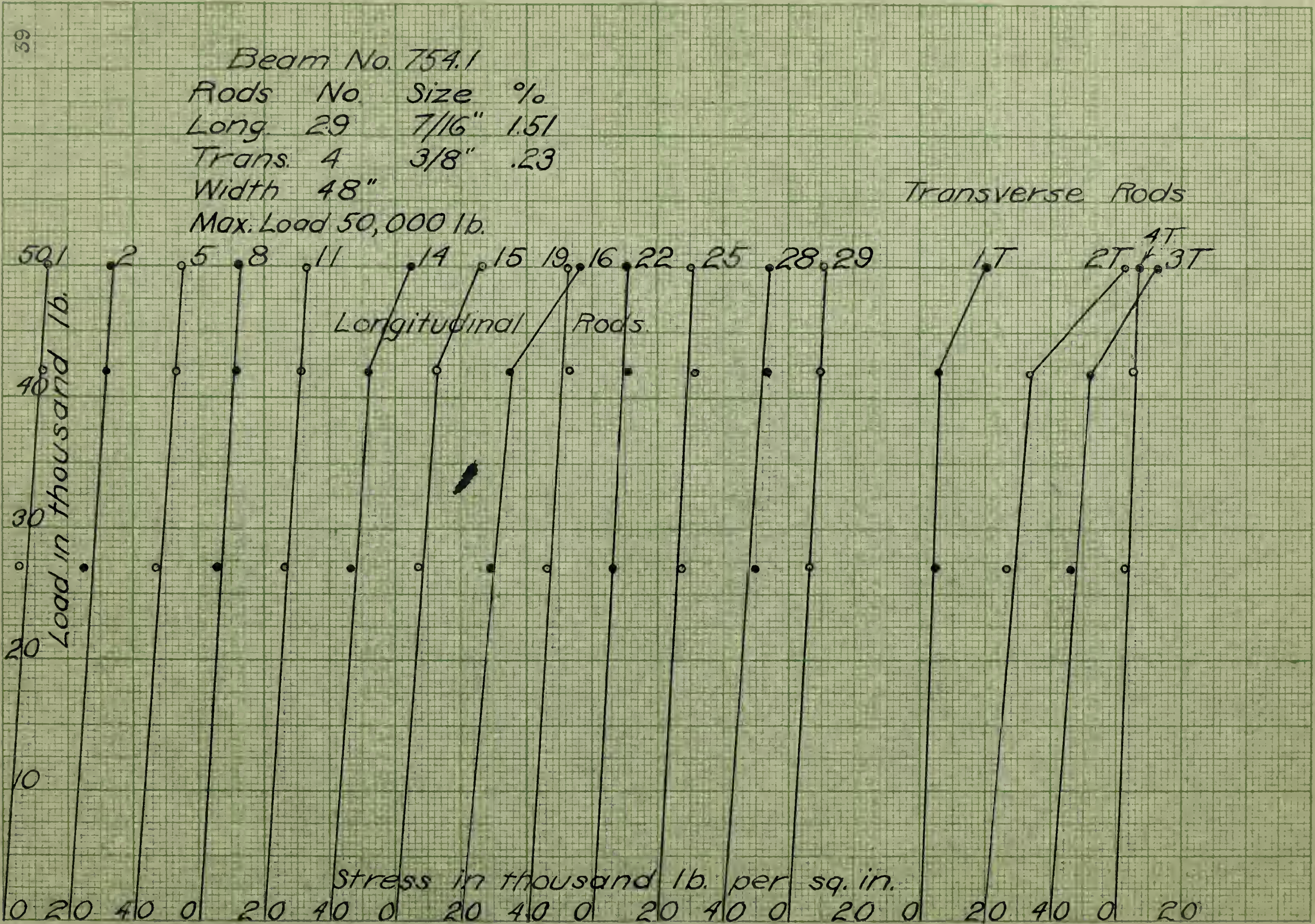
Rods	No.	Size	%
Long	29.	7/16"	1.51
Trans	0	0	0

Width 48"

Max. Load 42,000 lb.

Longitudinal Rods.



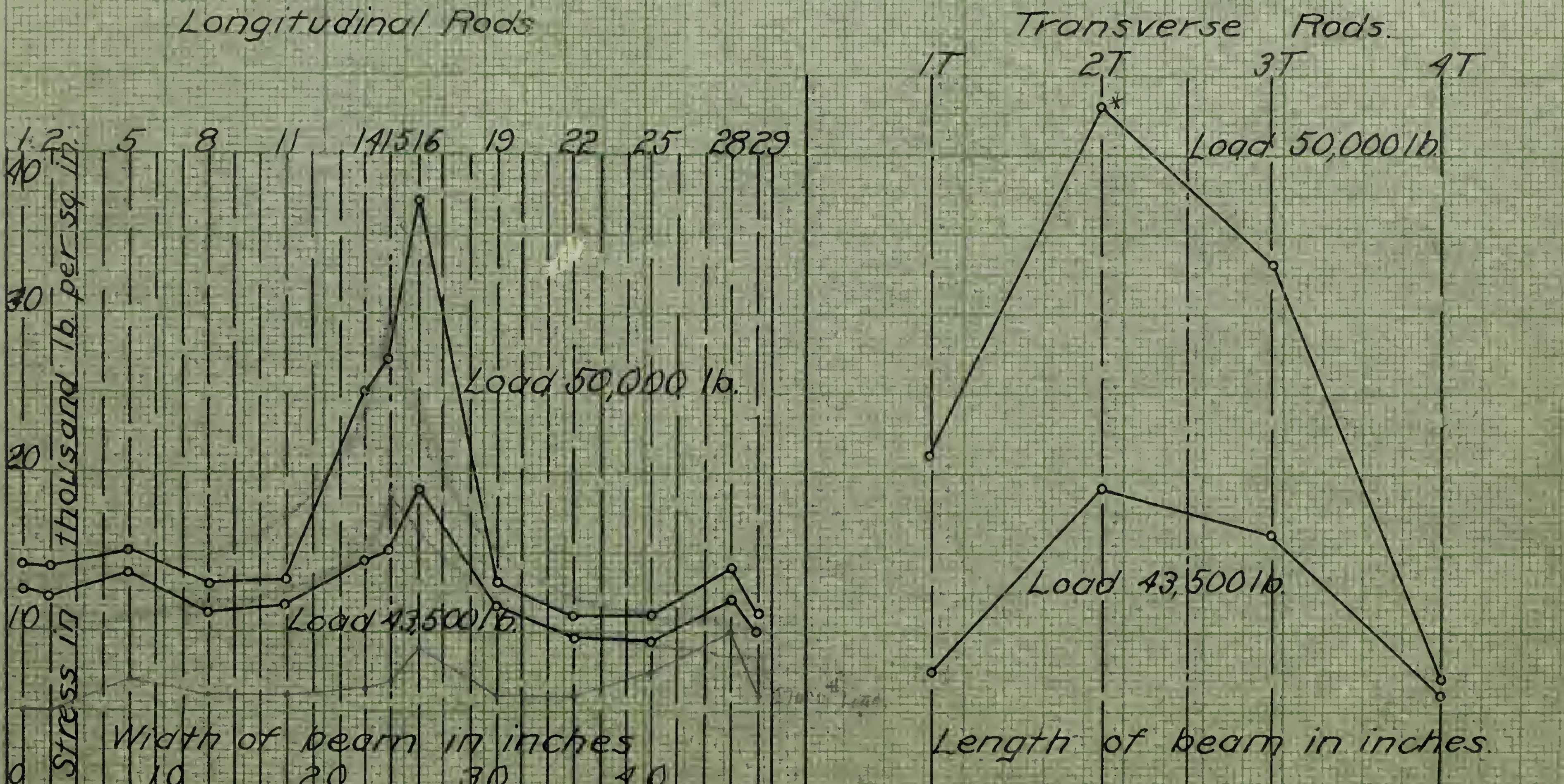

Beam No. 754.1
Rods No. Size %
Long. 29 7/16" 1.51
Trans. 4 3/8" .23
Width 48"
Max. Load 50,000 lb.
Transverse Rods
Longitudinal Rods
Load in thousand lb.
Stress in thousand lb. per sq. in.
1 2 5 8 11 14 15 19 16 22 25 28 29
1T 2T 4T 3T
50 40 30 20 10

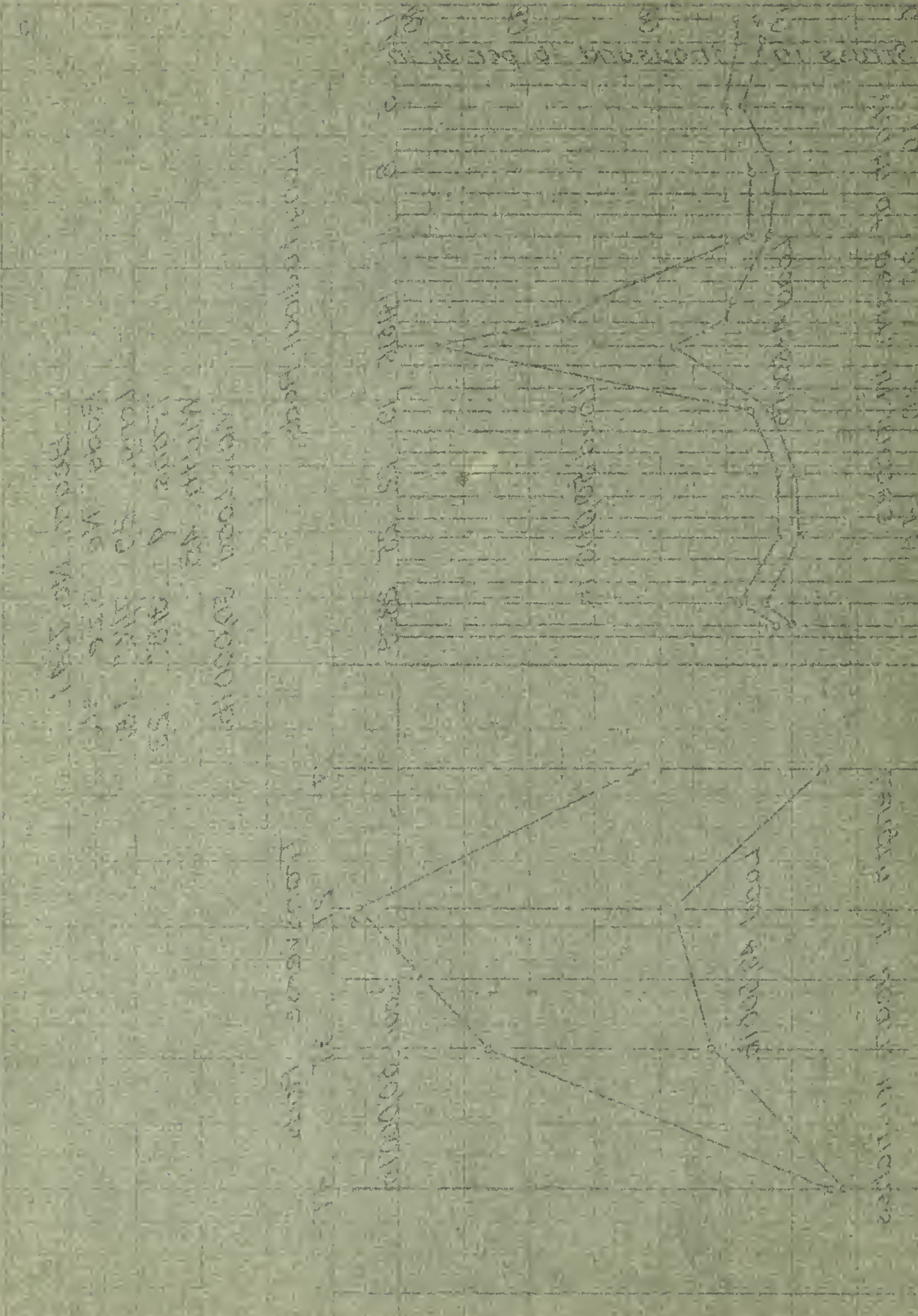
Beam No. 7541

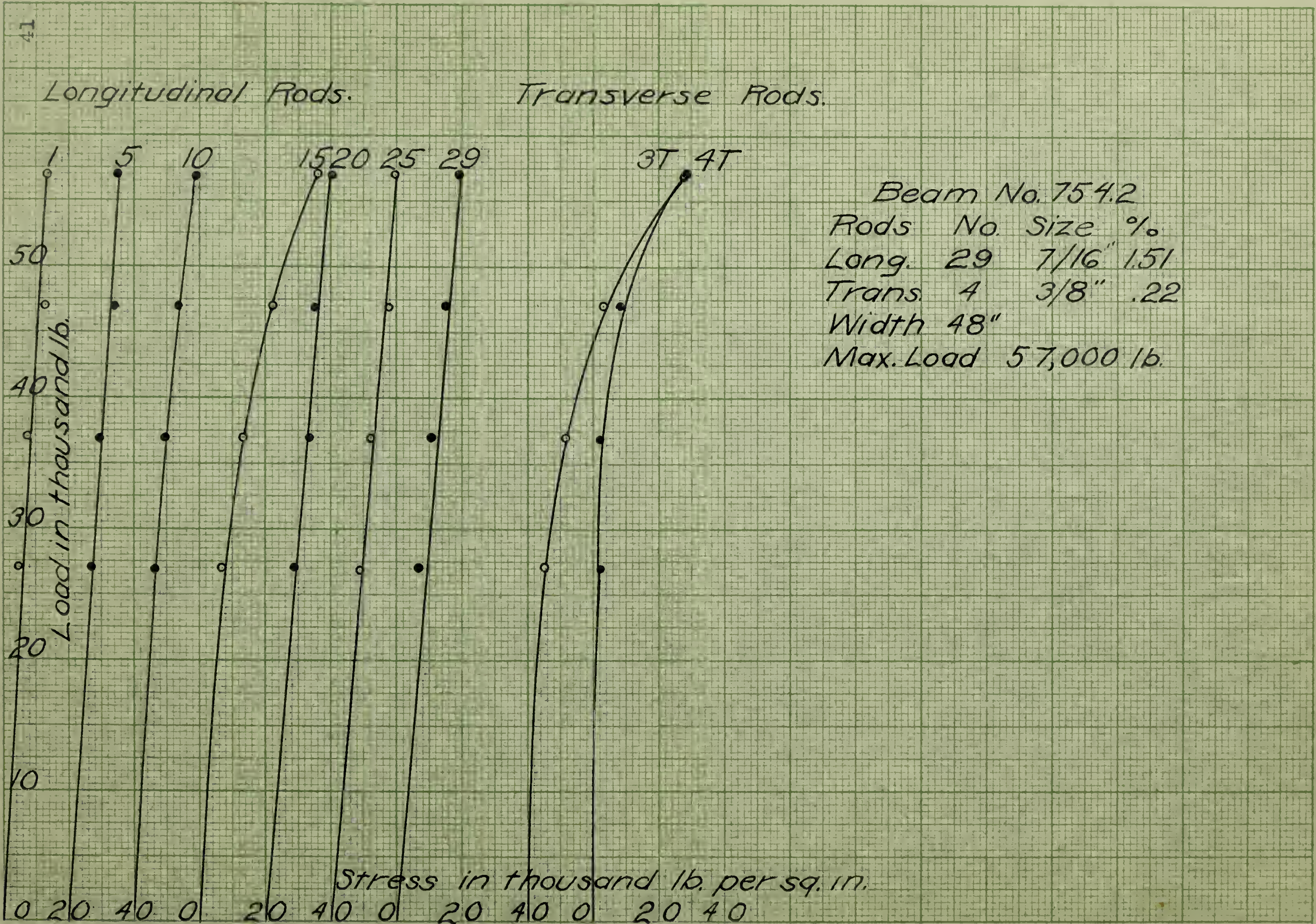
Rods	No.	Size	%
Long.	29	7/16"	1.51
Trans.	4	3/8"	.23

Width 48"

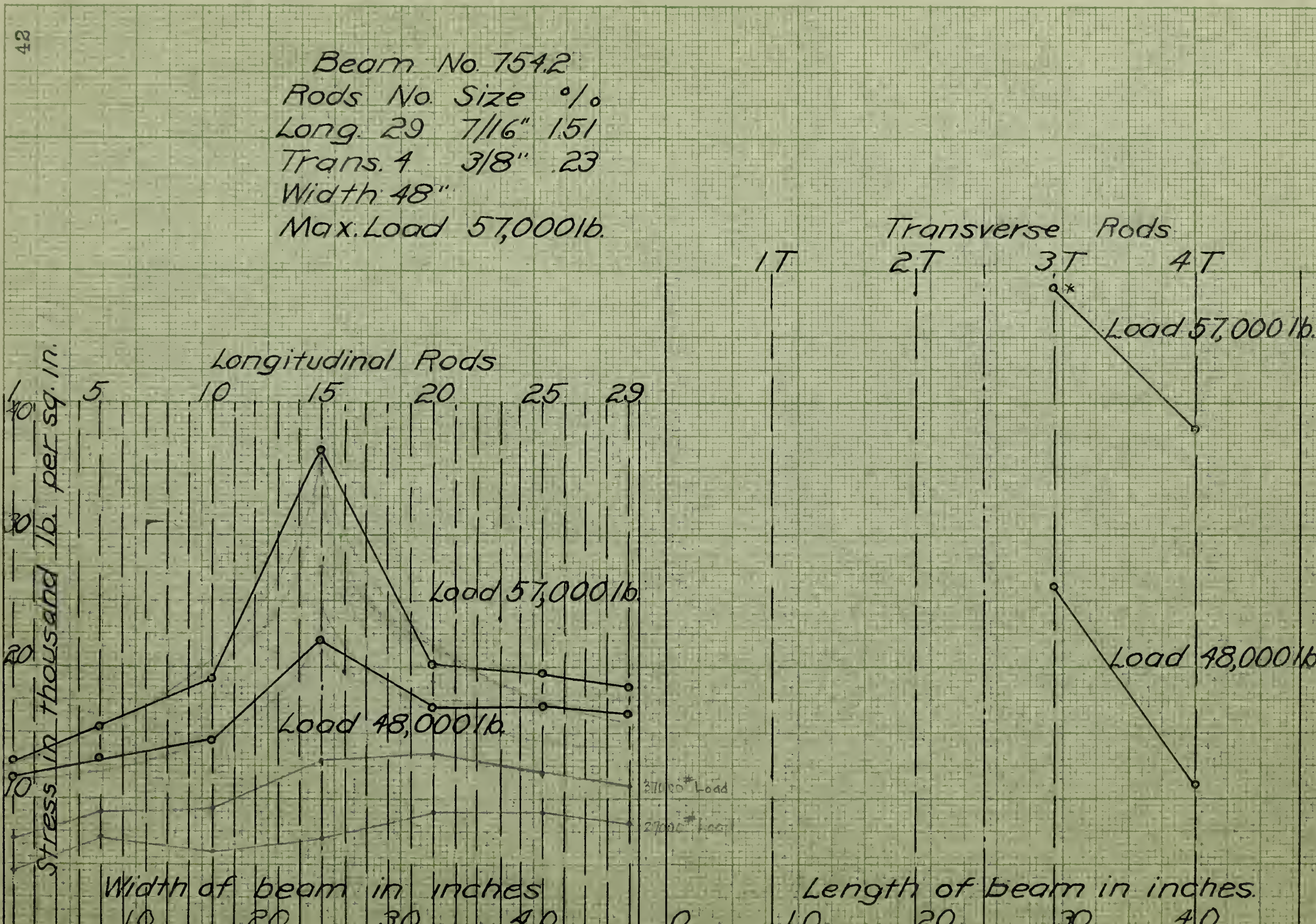
Max. Load 50,000 lb.

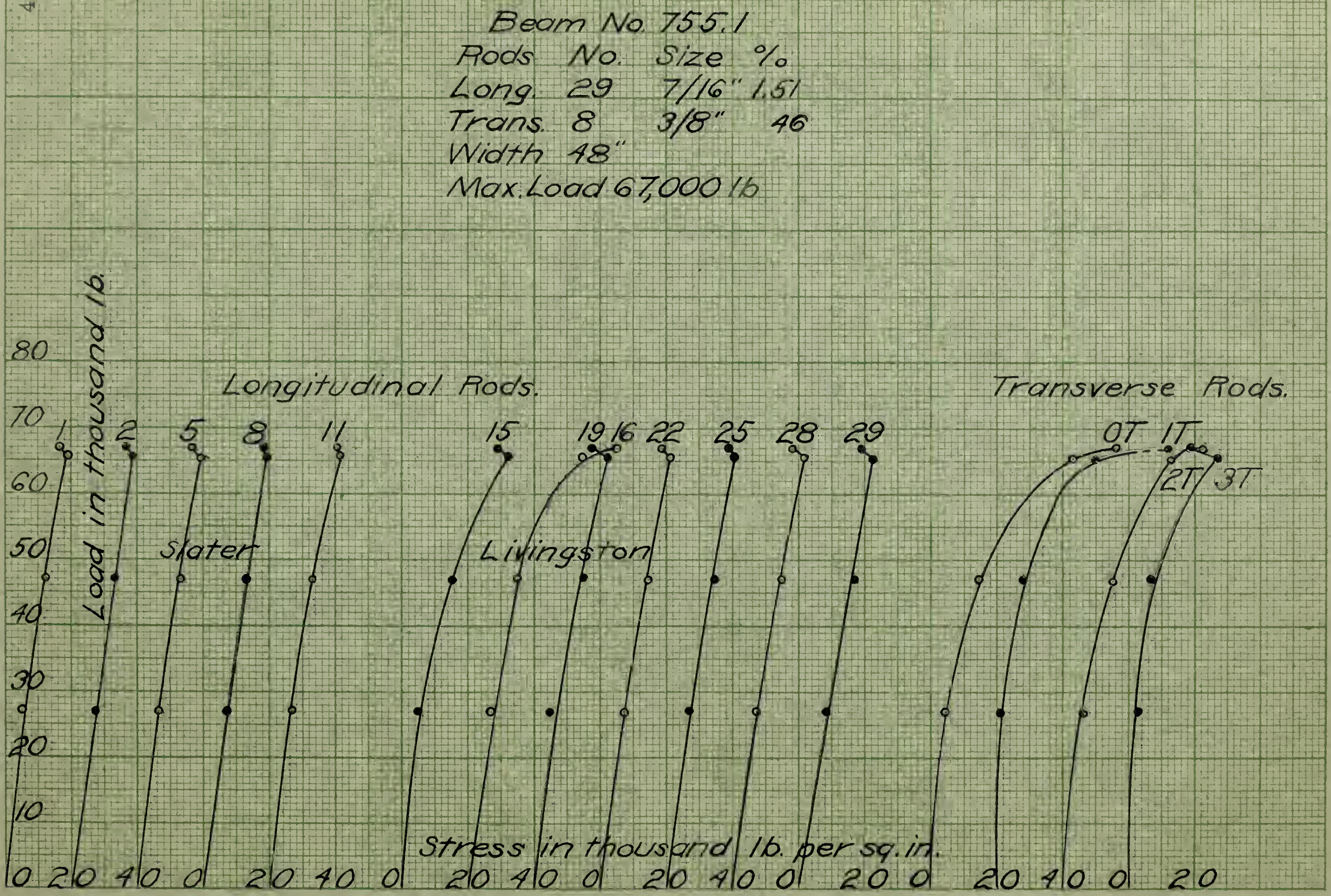






Longitudinal Rods.
Transverse Rods.
1
5
10
15
20
25
29
3T
4T
Beam No. 754.2
Rods No. Size %
Long. 29 7/16" 1.51
Trans. 4 3/8" .22
Width 48"
Max. Load 57,000 lb.
Load in thousand lb.
10
20
30
40
50
Stress in thousand lb. per sq. in.
0 20 40 0 20 40 0 20 40 0 20 40


Beam No. 754.2
Rods No. Size %
Long. 29 7/16" 1.51
Trans. 4 3/8" .23
Width 48"
Max. Load 57,000 lb.
Longitudinal Rods
1 5 10 15 20 25 29
Stress in thousand lb. per sq. in.
10 20 30 40
Load 57,000 lb.
Load 48,000 lb.
37000# Load
27000# Load
Width of beam in inches
10 20 30 40
Transverse Rods
1T 2T 3T 4T
Load 57,000 lb.
Load 48,000 lb.
Length of beam in inches.
0 10 20 30 40

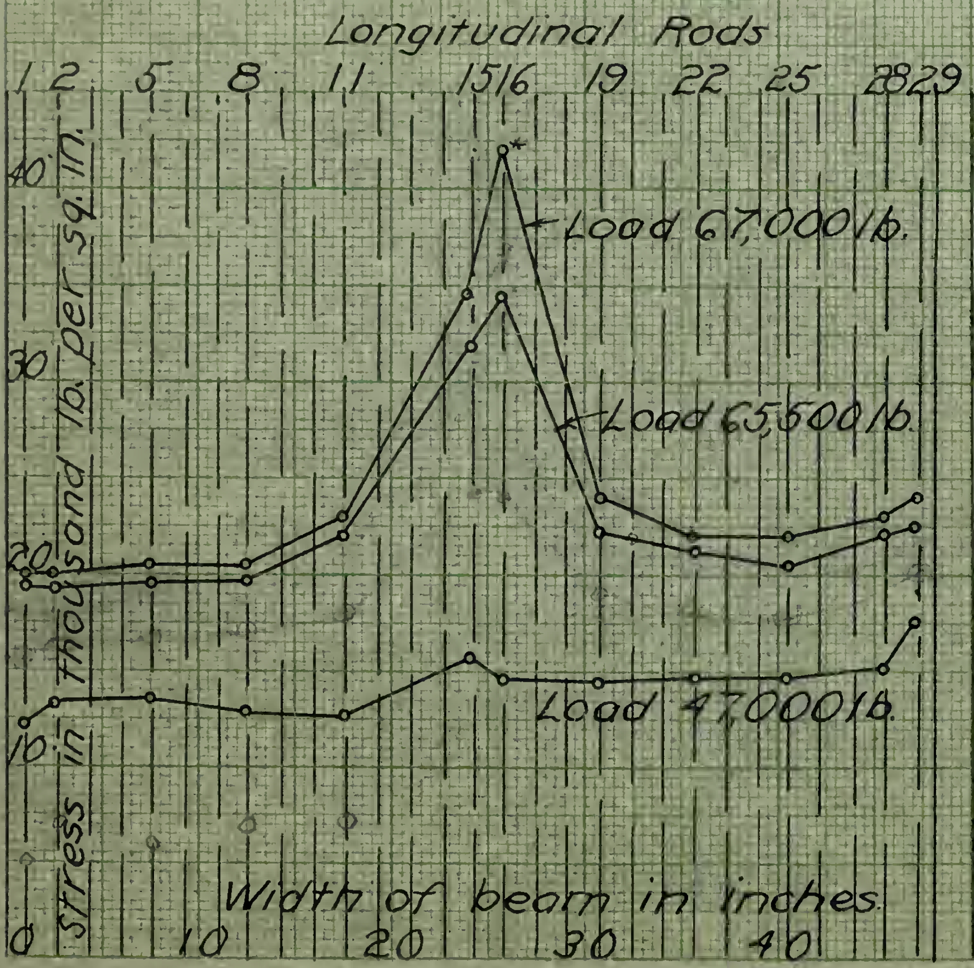

Beam No. 755.1
Rods No. Size %
Long. 29 7/16" 1.51
Trans. 8 3/8" 46
Width 48"
Max. Load 67,000 lb
Load in thousand lb.
80
70
60
50
40
30
20
10
Longitudinal Rods.
Transverse Rods.
1 2 5 8 11 15 19 16 22 25 28 29
0T 1T 2T 3T
Slater
Livingston
Stress in thousand lb. per sq. in.
0 20 40 0 20 40 0 20 40 0 20 40 0 20 0 20 40 0 20

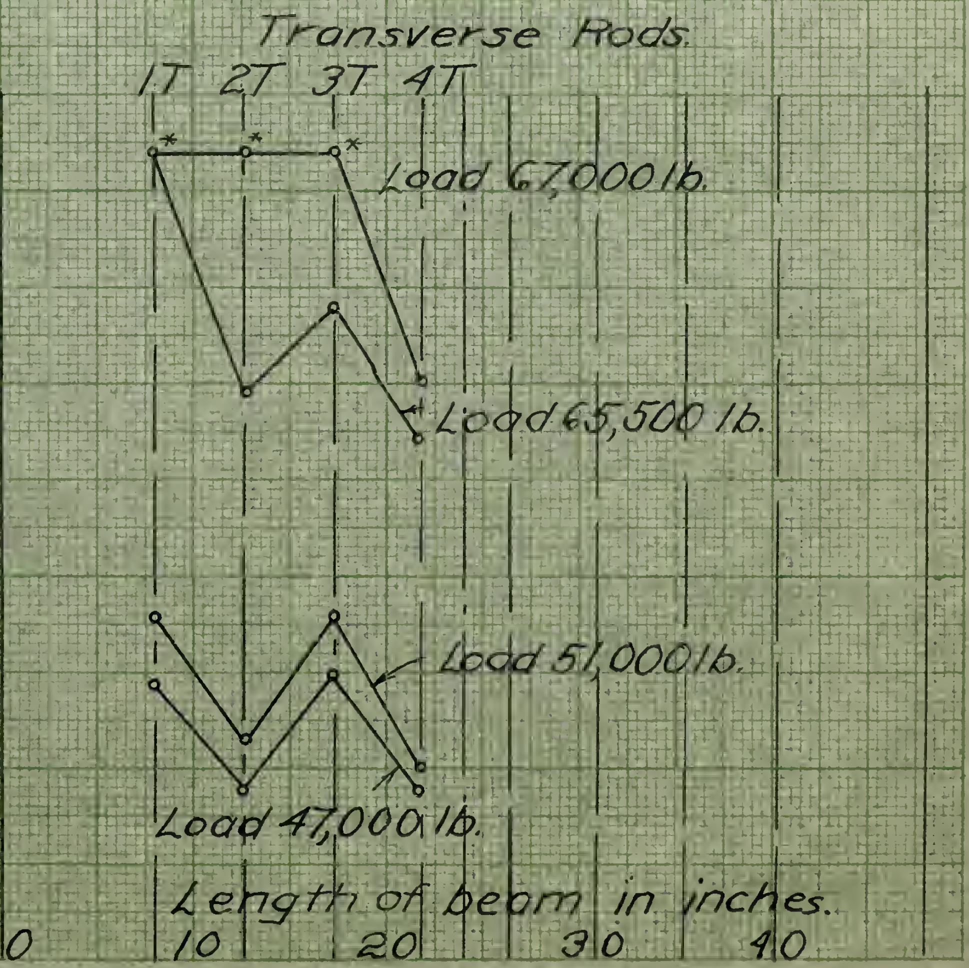
Beam No. 755.1

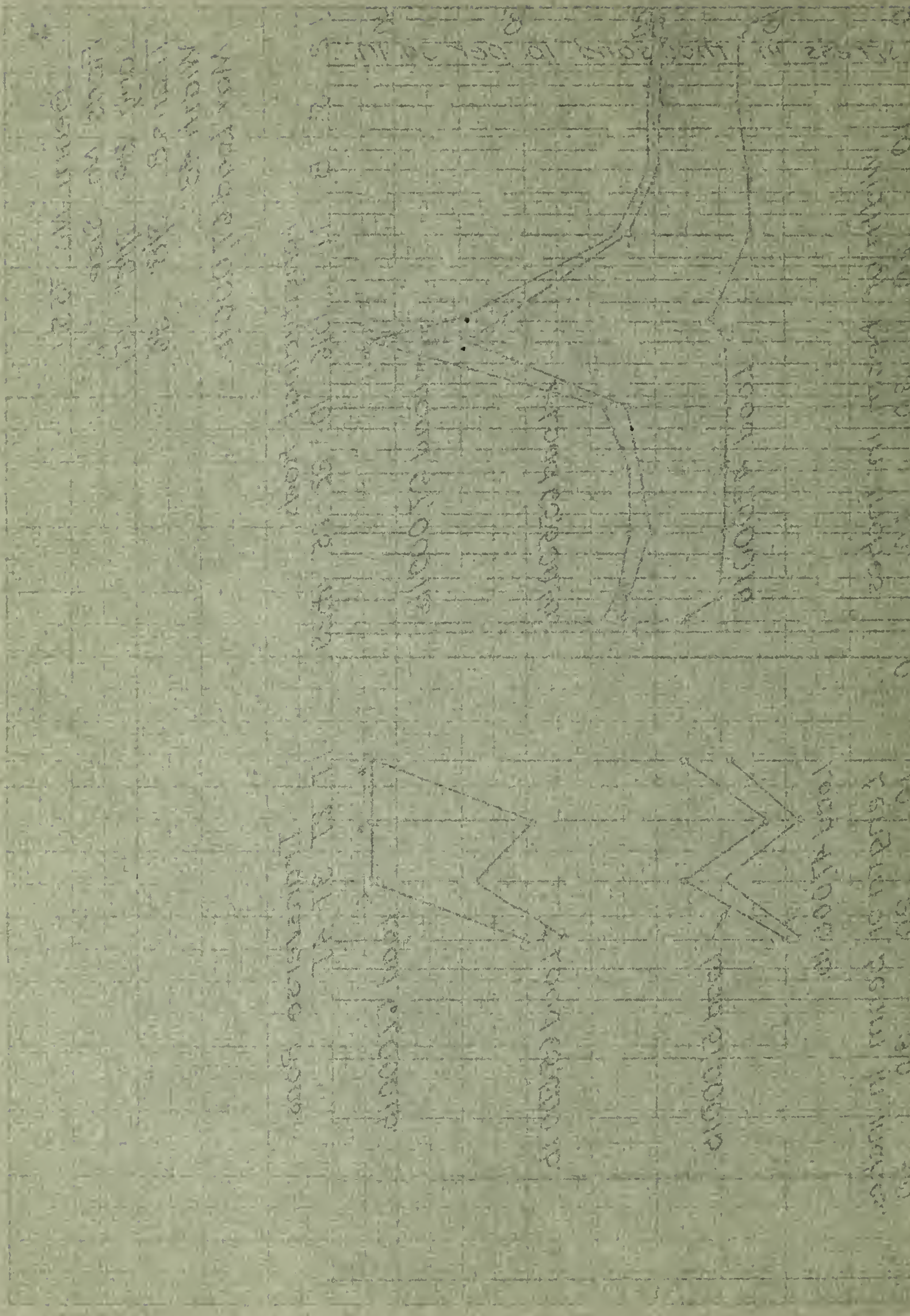
Rods	No.	Size	%
Long.	29	7/16"	1.51
Trans.	8	3/8"	.46

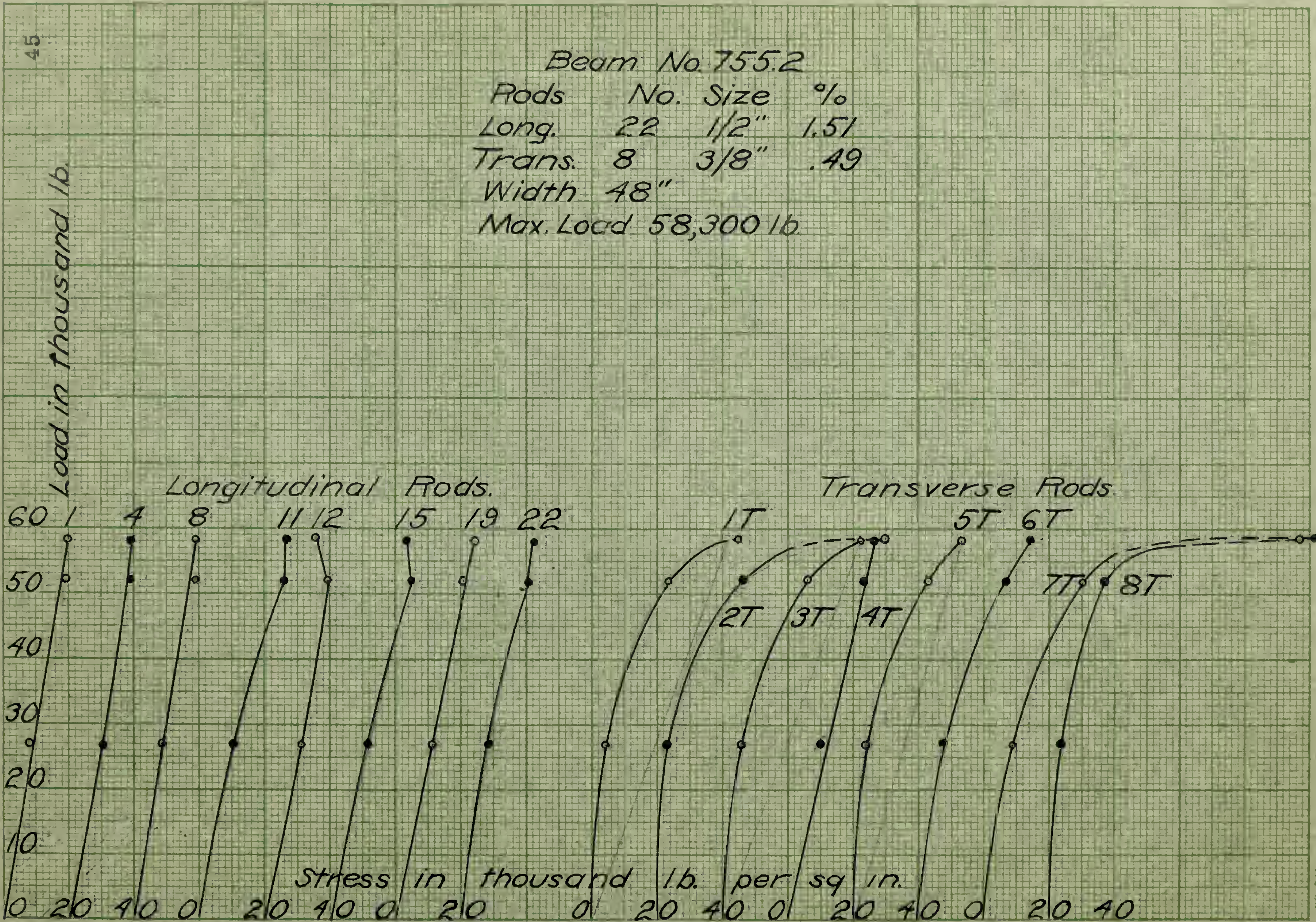
Width 48"

Max. Load 67,000 lb.

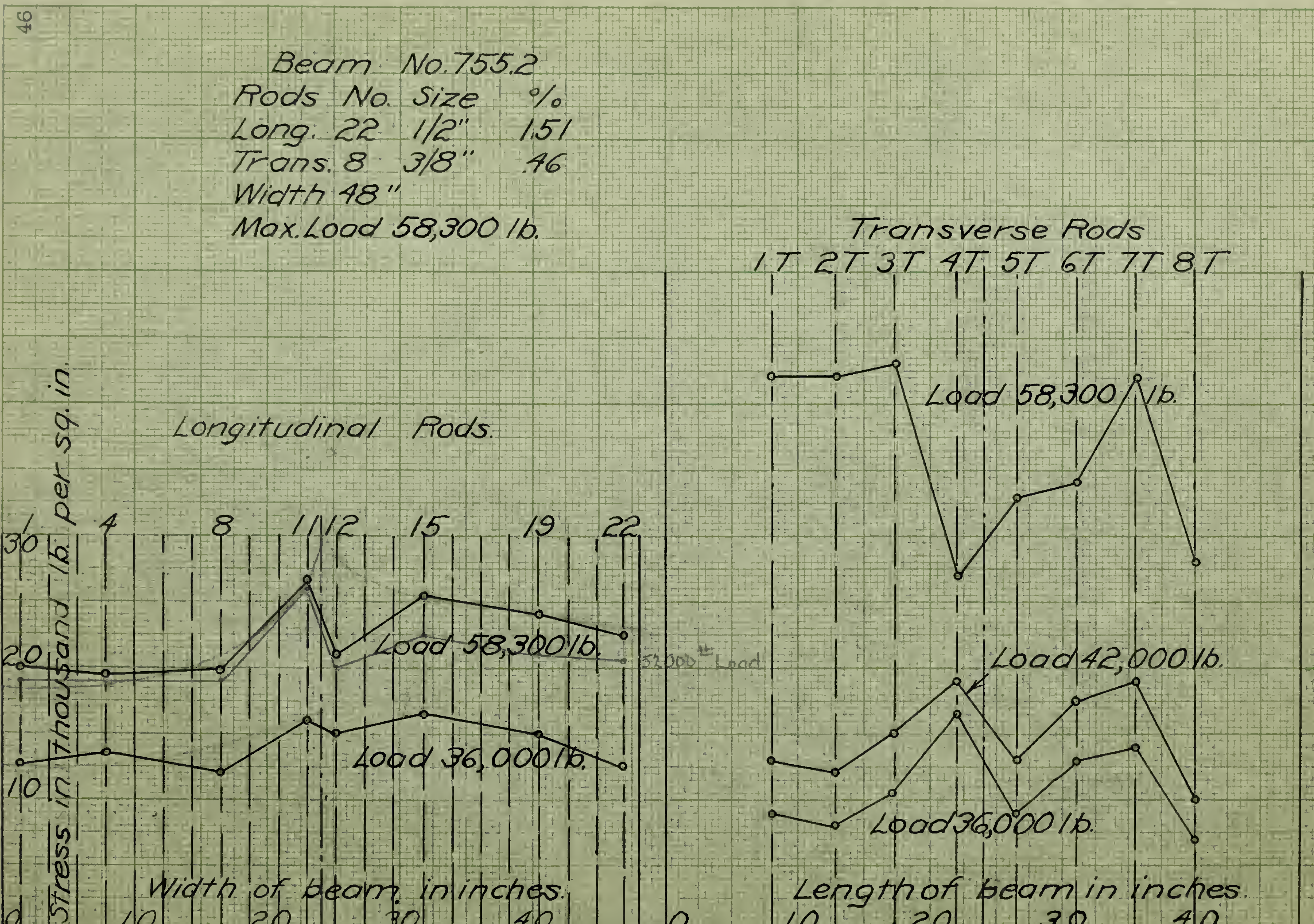








Beam No 755.2
Rods No. Size %
Long. 22 1/2" 1.51
Trans. 8 3/8" .49
Width 48"
Max. Load 58,300 lb.
Load in thousand lb.
Longitudinal Rods.
Transverse Rods
60 1 4 8 11 12 15 19 22
1T 2T 3T 4T 5T 6T 7T 8T
50
40
30
20
10
0 20 40 0 20 40 0 20 0 20 40 0 20 40 0 20 40
Stress in thousand lb. per sq in.

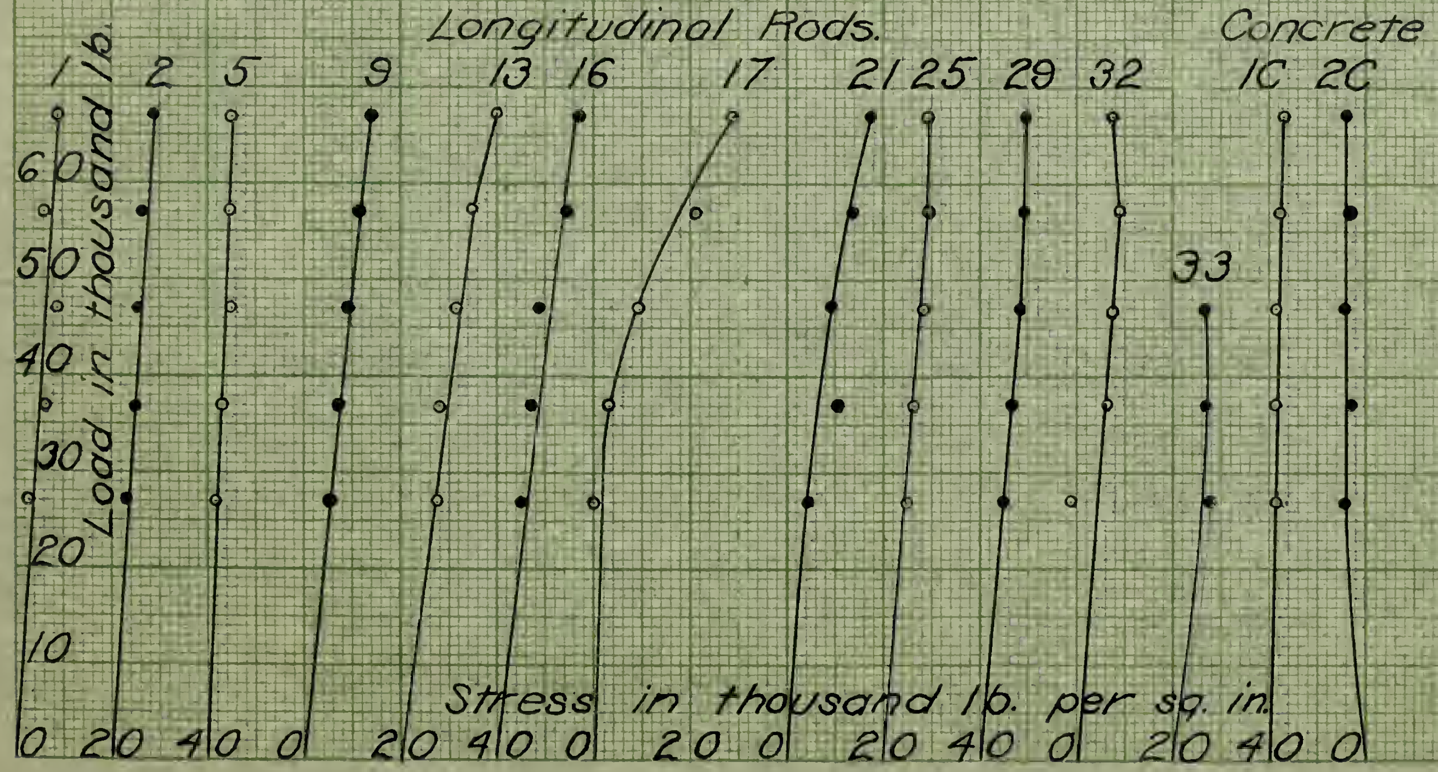

Beam No. 755.2
Rods No. Size %
Long. 22 1/2" 1.51
Trans. 8 3/8" .46
Width 48"
Max. Load 58,300 lb.
Longitudinal Rods.
1 4 8 11 12 15 19 22
30
20
10
0
Load 58,300 lb.
Load 36,000 lb.
52,000 lb Load
Stress in thousand lb. per sq. in.
Width of beam in inches.
10 20 30 40
Transverse Rods
1T 2T 3T 4T 5T 6T 7T 8T
Load 58,300 lb.
Load 42,000 lb.
Load 36,000 lb.
Length of beam in inches.
0 10 20 30 40

Beam No. 756.1

Rods	No.	Size	%
Long.	33	1/2"	1.5
Trans.	4	3/8"	.23

Width 72"

Max Load 68,000 lb.



Beam No. 756.1

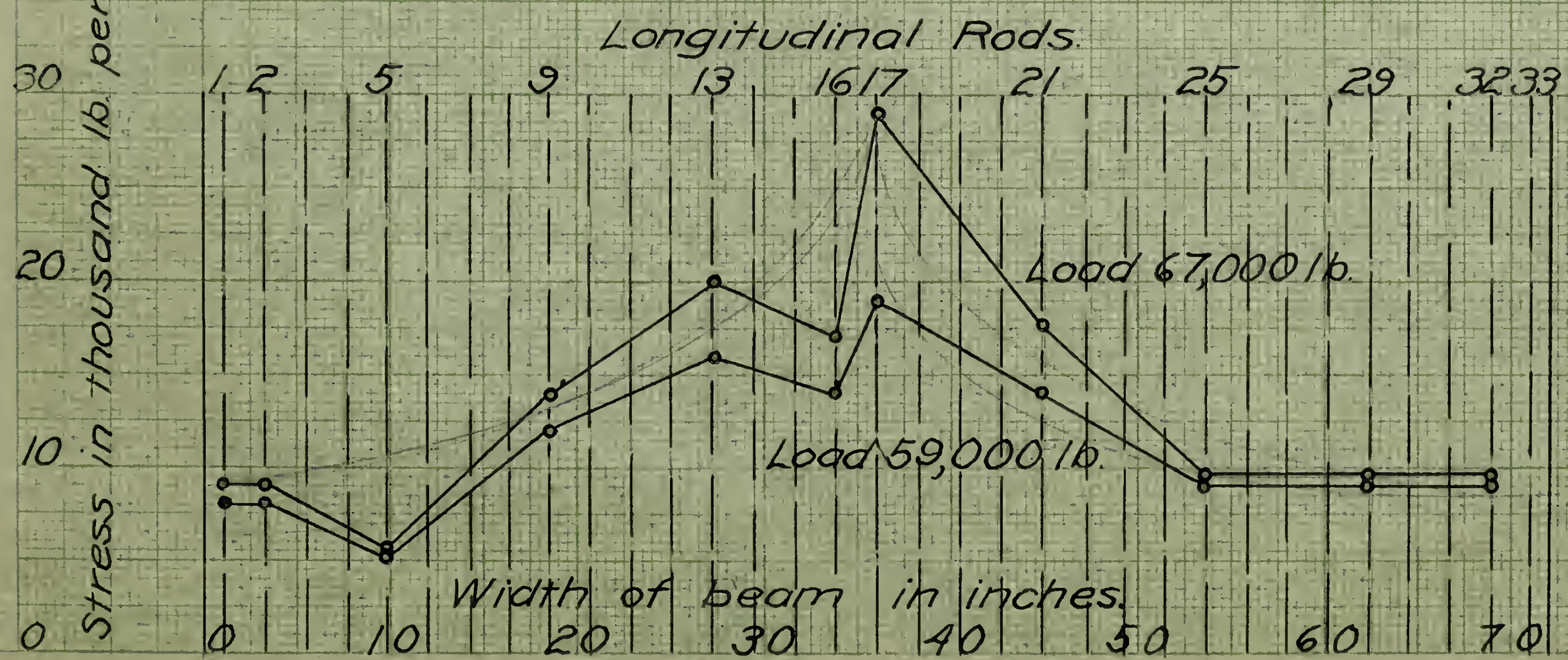
Rods No. Size %

Long. 33 1/2" 1.50

Trans. 4 3/8" .23

Width 72"

Max. Load 68,000 lb.

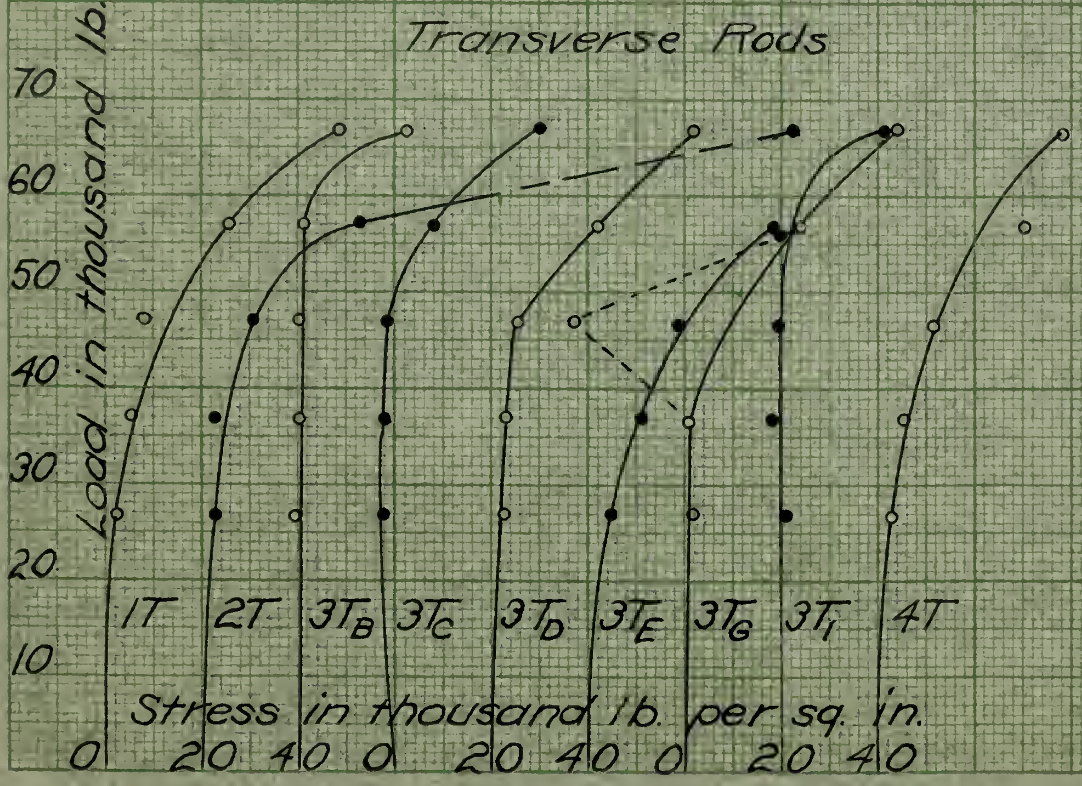


Beam No. 756.1

Rods	No.	Size	%
Long.	33	1/2"	1.5
Trans.	4	3/8"	.23

Width 72"

Max. Load 68,000 lb.

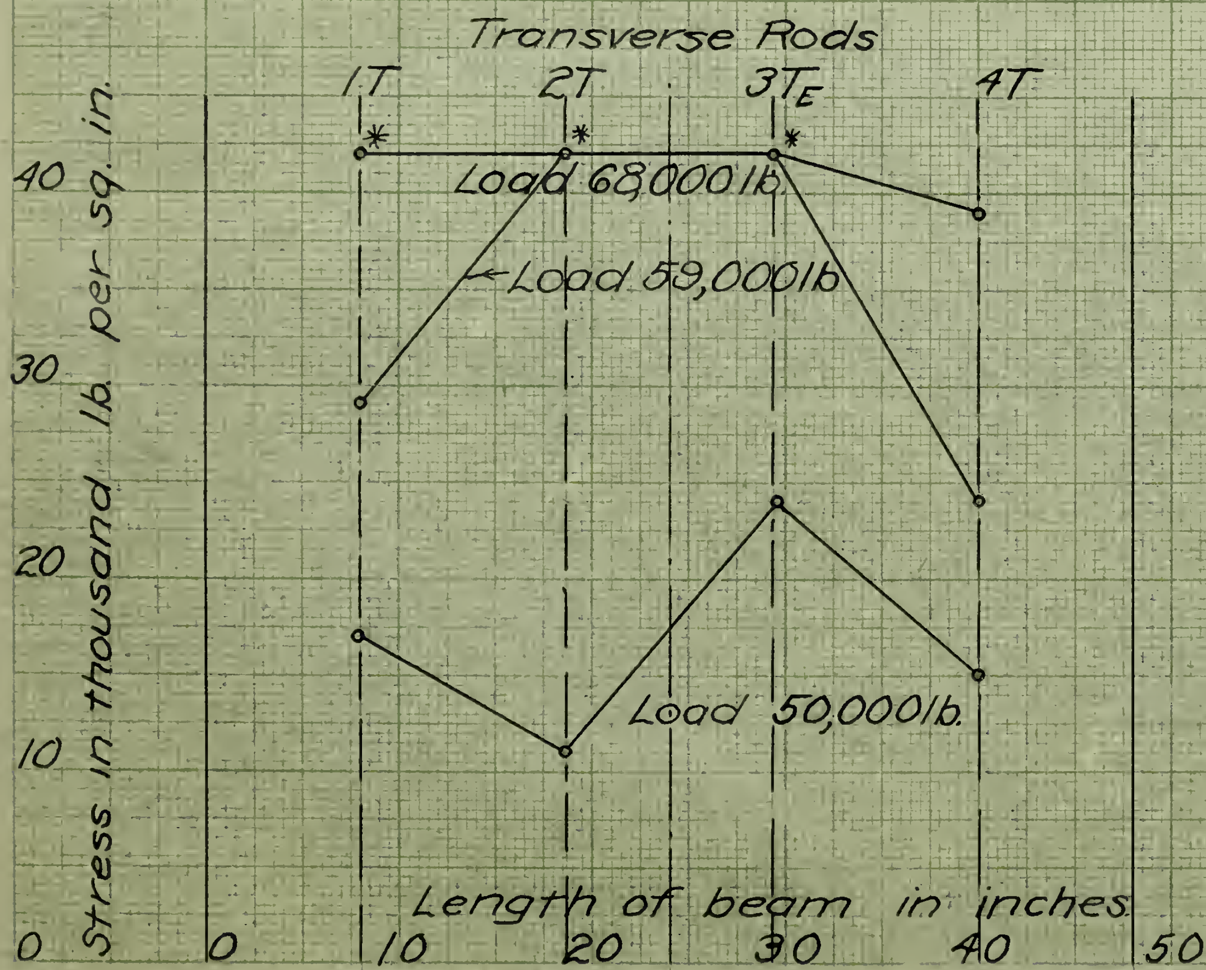


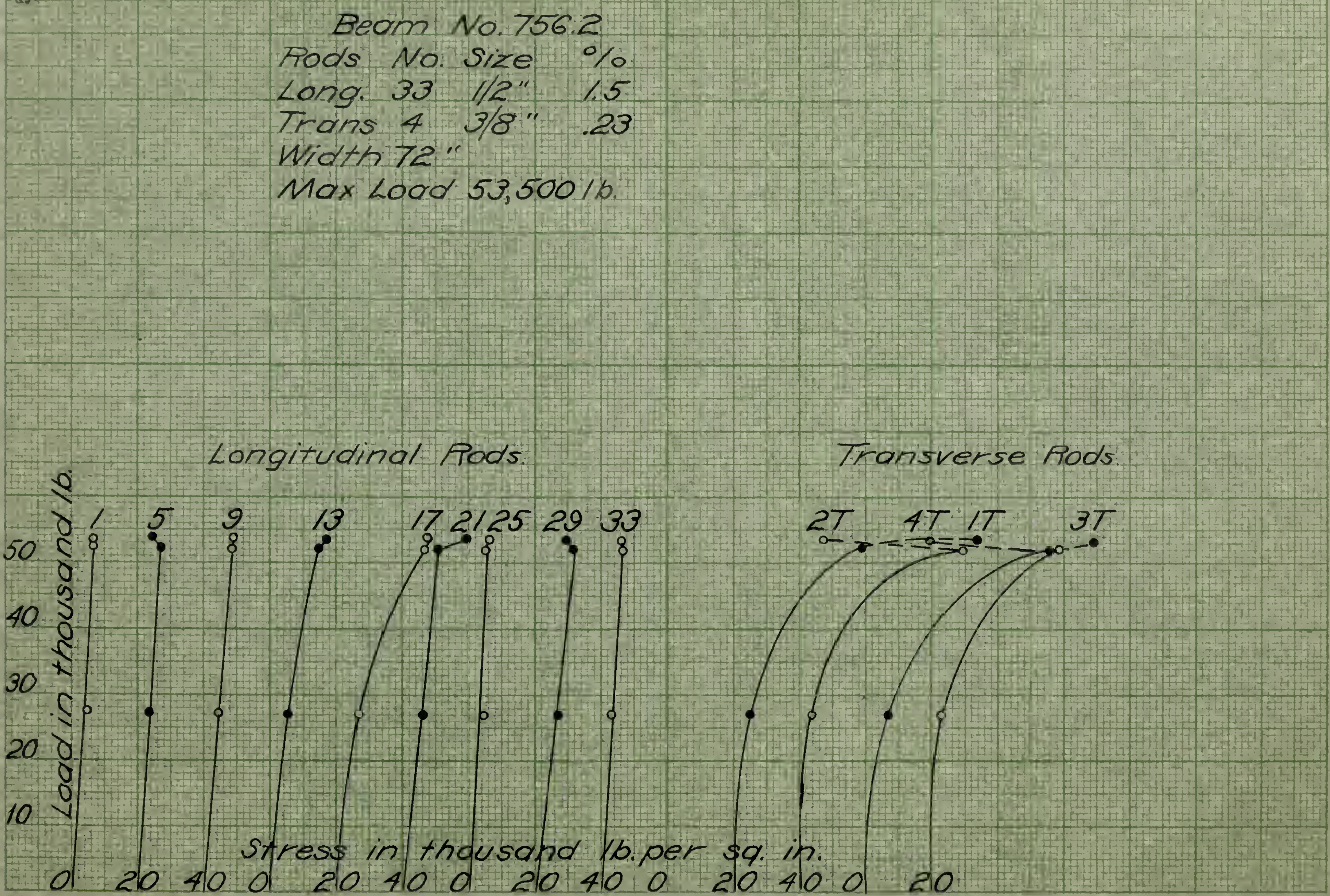
Beam No. 756.1

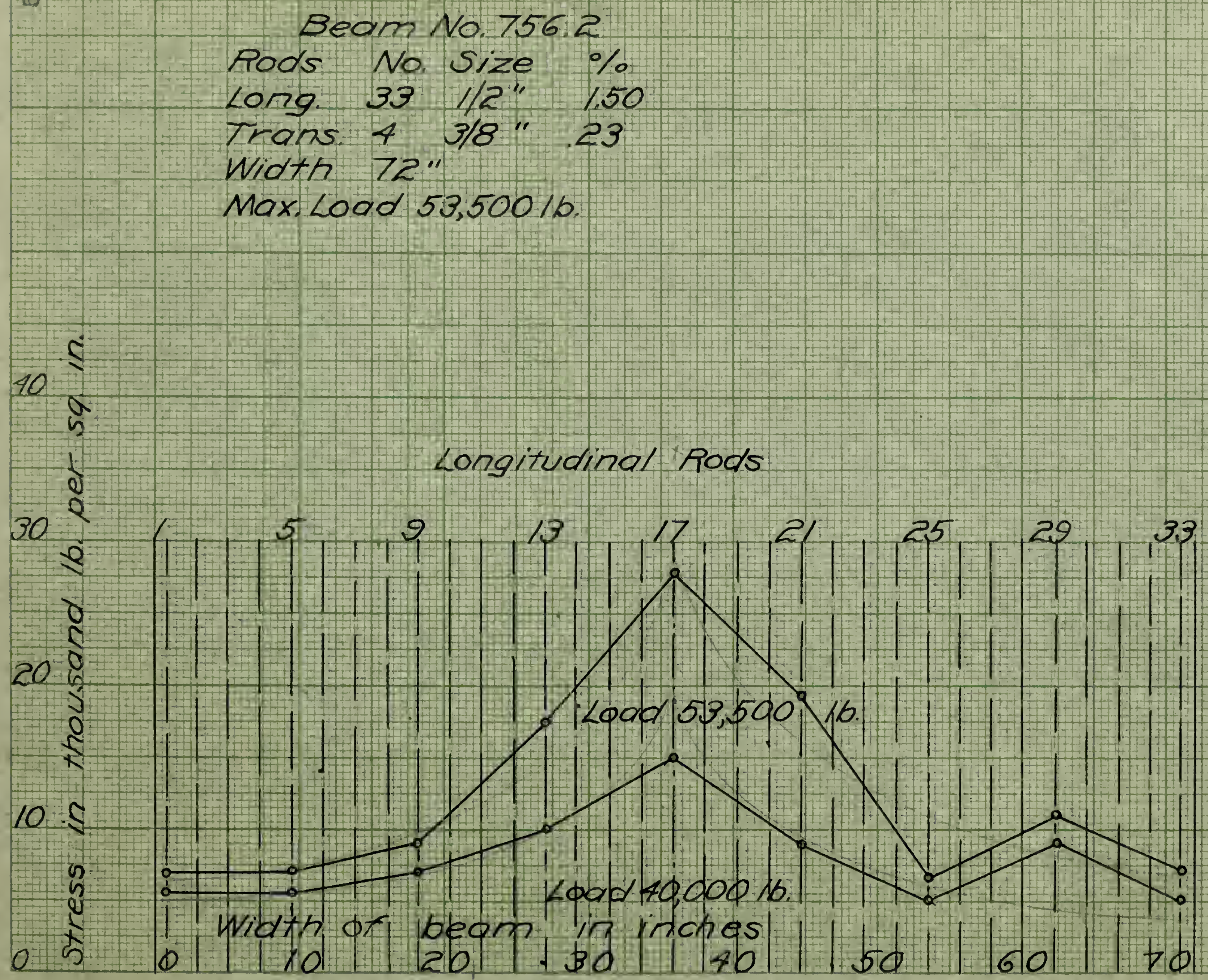
Rods	No.	Size	%
Long.	33	1/2"	1.50
Trans	4	3/8"	.23

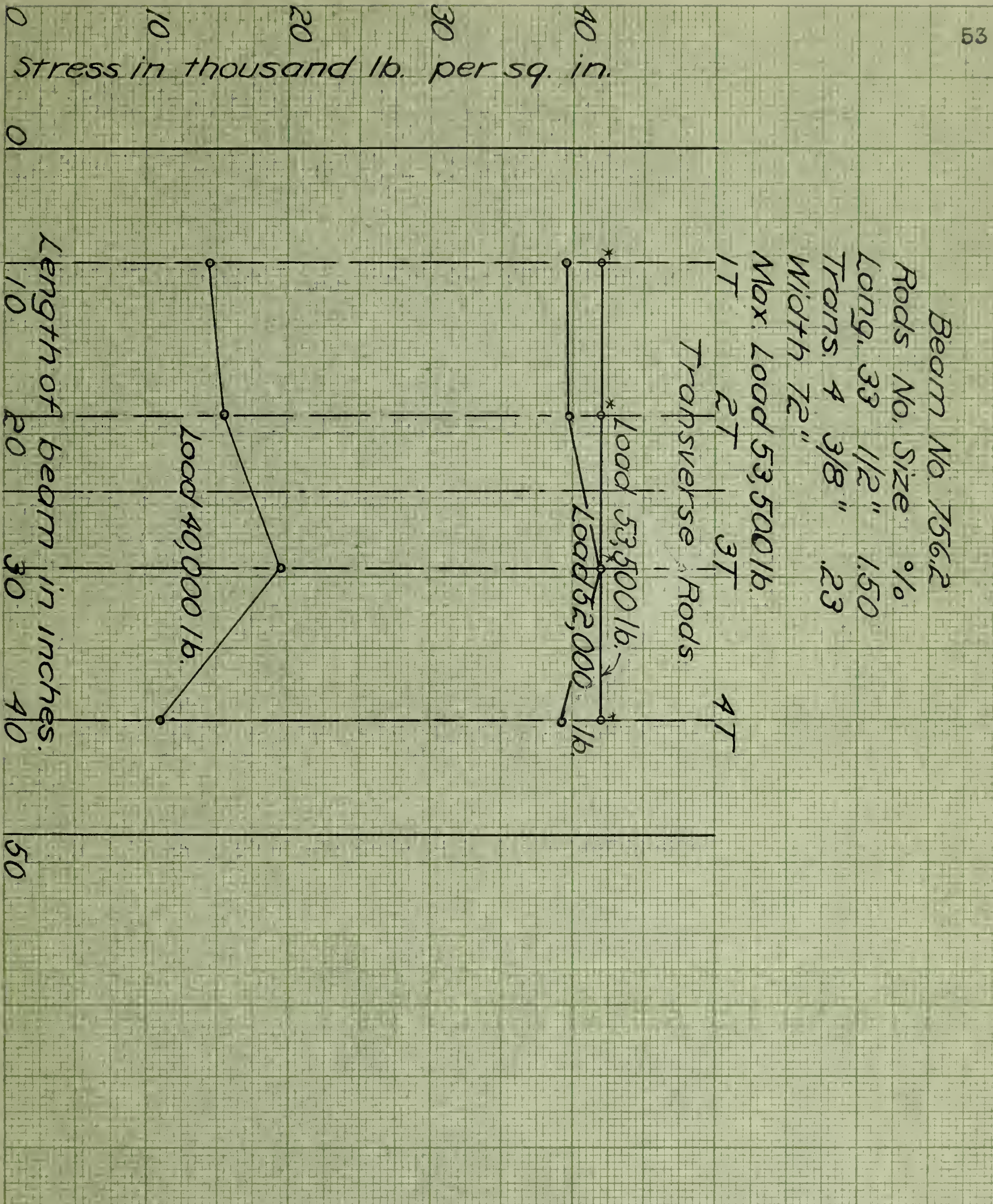
Width 72"

Max. Load 68,000 lb.



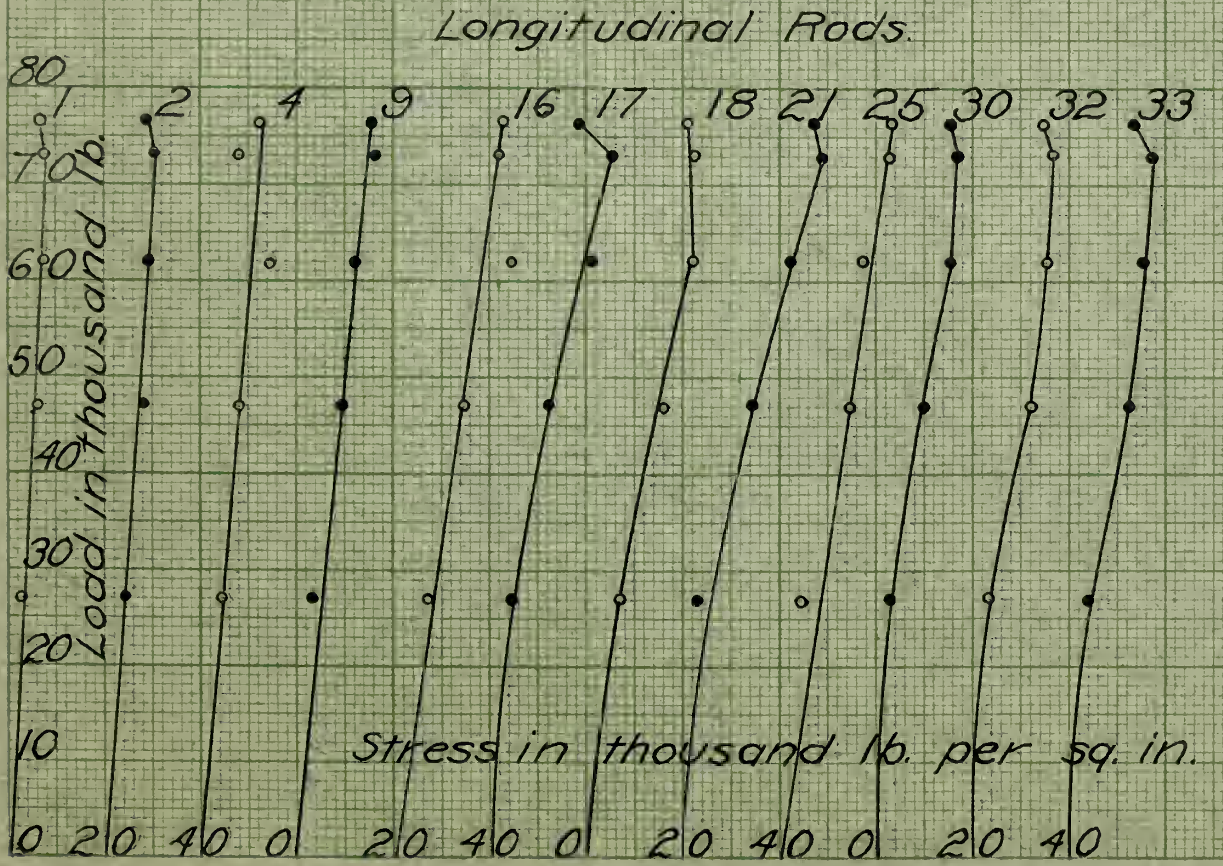

Beam No. 756.2
Rods No. Size %
Long. 33 1/2" 1.5
Trans 4 3/8" .23
Width 72"
Max Load 53,500 lb.
Longitudinal Rods.
Transverse Rods.
Load in thousand lb.
1 5 9 13 17 21 25 29 33
2T 4T 1T 3T
Stress in thousand lb. per sq. in.
0 20 40 0 20 40 0 20 40 0 20 40 0 20
10 20 30 40 50

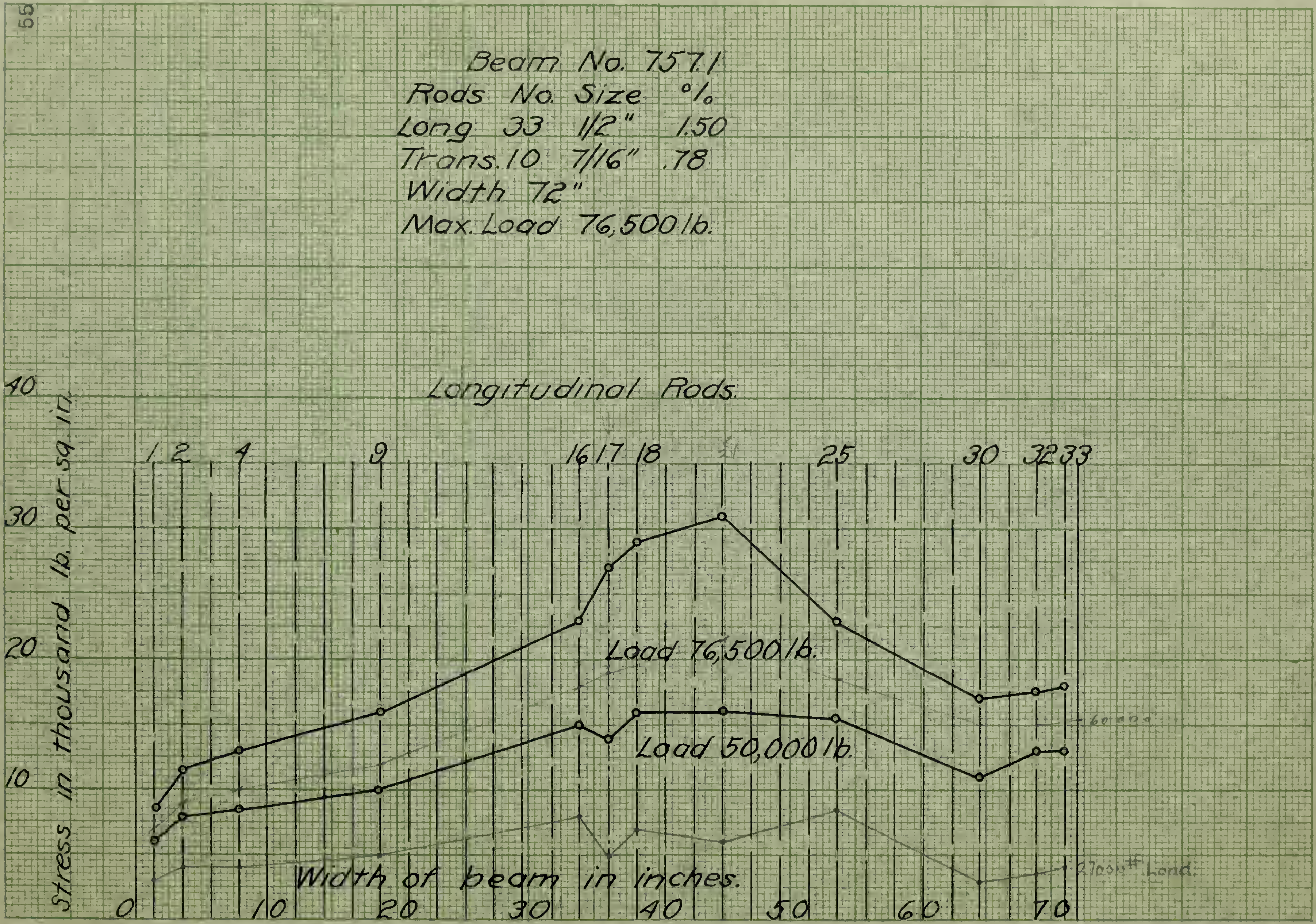

Beam No. 756.2
Rods No. Size %
Long. 33 1/2" 1.50
Trans. 4 3/8" .23
Width 72"
Max. Load 53,500 lb.
Longitudinal Rods
1 5 9 13 17 21 25 29 33
Load 53,500 lb.
Load 40,000 lb.
Stress in thousand lb. per sq. in.
0 10 20 30 40
Width of beam in inches
0 10 20 30 40 50 60 70


Stress in thousand lb. per sq. in.
0
10
20
30
40
Beam No. 756.2
Rods No. Size %
Long. 33 1/2" 1.50
Trans. 4 3/8" .23
Width 72"
Max. Load 53,500 lb.
Transverse Rods.
1T
2T
3T
4T
Load 53,500 lb.
Load 52,000 lb.
Load 40,000 lb.
Length of beam in inches.
0
10
20
30
40
50

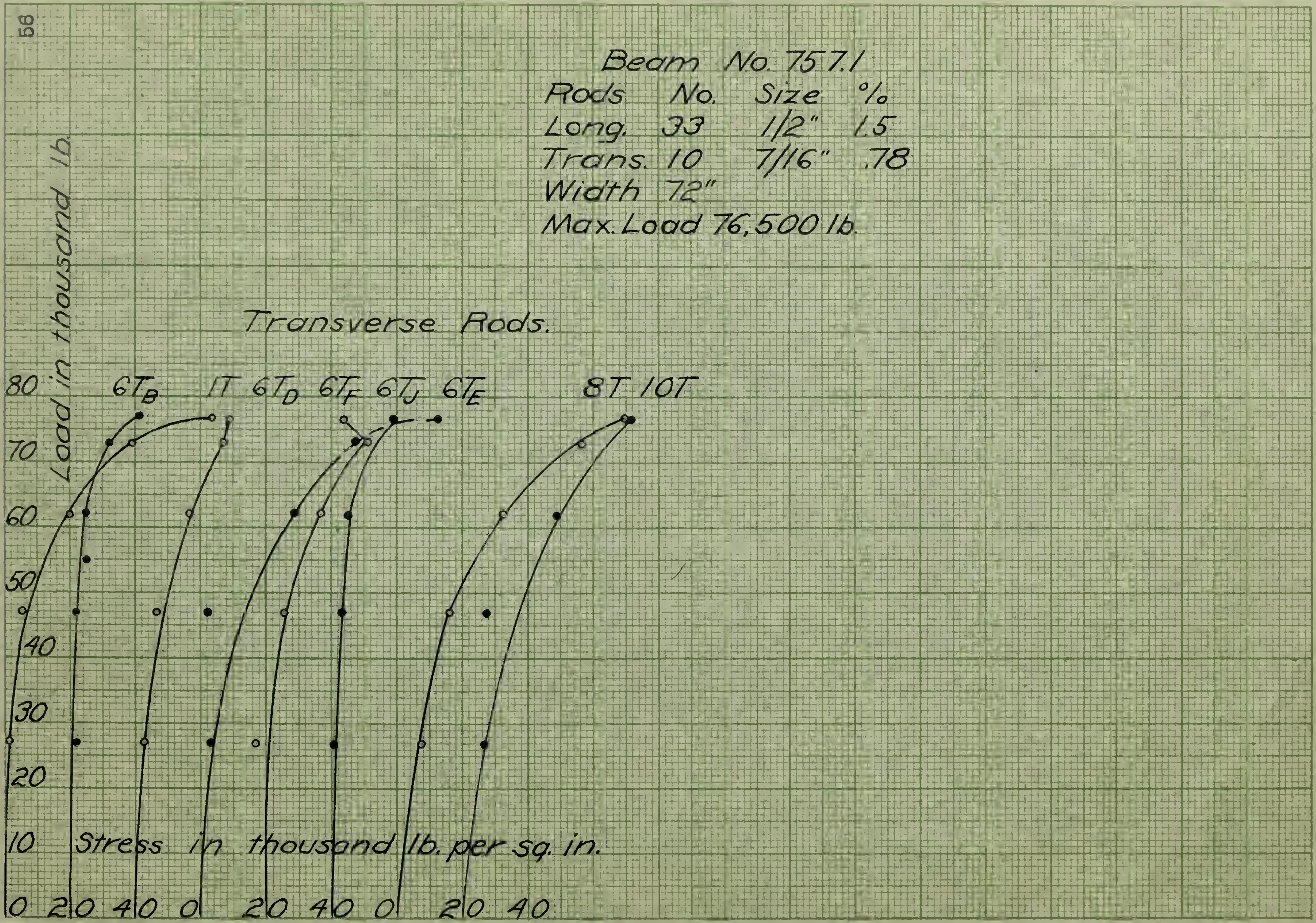
Beam No. 757.1

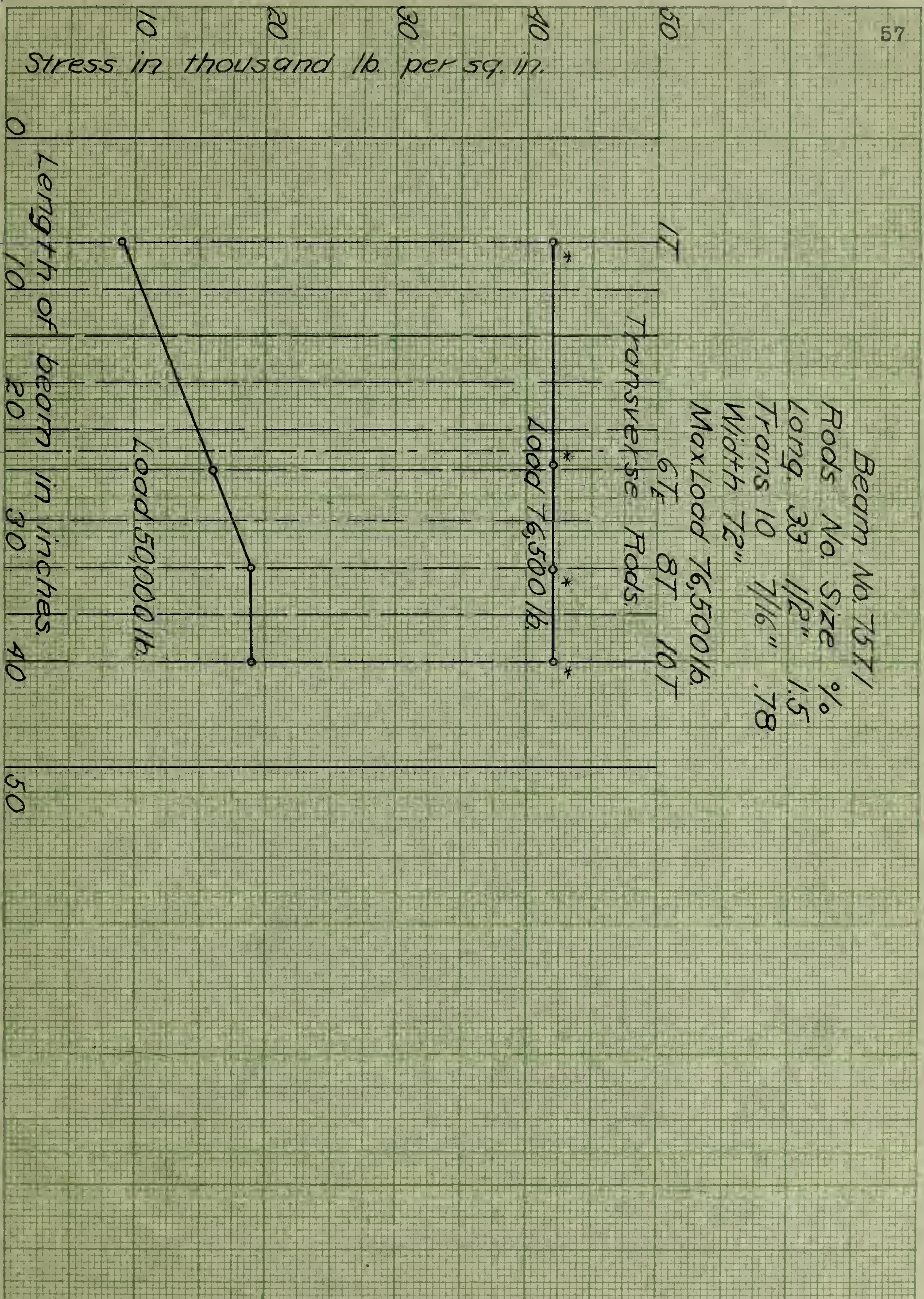
Rods	No.	Size	%
Long.	33	1/2"	1.5
Trans.	10	7/16"	.78
Width	72"		
Max. Load 76,500 lb.			





Beam No. 757.1
Rods No. Size %
Long 33 1/2" 1.50
Trans. 10 7/16" .78
Width 72"
Max. Load 76,500 lb.
Longitudinal Rods.
1 2 4 9 16 17 18 25 30 32 33
Load 76,500 lb.
Load 50,000 lb.
Stress in thousand lb. per sq. in.
0 10 20 30 40
Width of beam in inches.
0 10 20 30 40 50 60 70


Beam No. 757.1
Rods No. Size %
Long. 33 1/2" 1.5
Trans. 10 7/16" .78
Width 72"
Max. Load 76,500 lb.
Transverse Rods.
Load in thousand lb.
$6T_B$ 1T $6T_D$ $6T_F$ $6T_J$ $6T_E$ 8T 10T
Stress in thousand lb. per sq. in.
0 10 20 30 40 50 60 70 80
0 20 40 0 20 40 0 20 40


Stress in thousand lb. per sq. in.
0 10 20 30 40 50
Length of beam in inches.
0 10 17 20 30 40 50
Load 50,000 lb.
Load 76,500 lb.
Transverse Rods.
6T 8T 10T
Beam No. 7571
Rods No. Size %
Long. 33 1/2" 1.5
Trans 10 7/16" .78
Width 72"
Max Load 76,500 lb.

Beam No. 757.2

Rods	No.	Size	%
Long.	33	1/2"	1.50
Trans.	13	3/8"	.75

Width 72"

Max. Load 82,000 lb.